THE OXFORD INSTITUTE FOR ENERGY STUDIES

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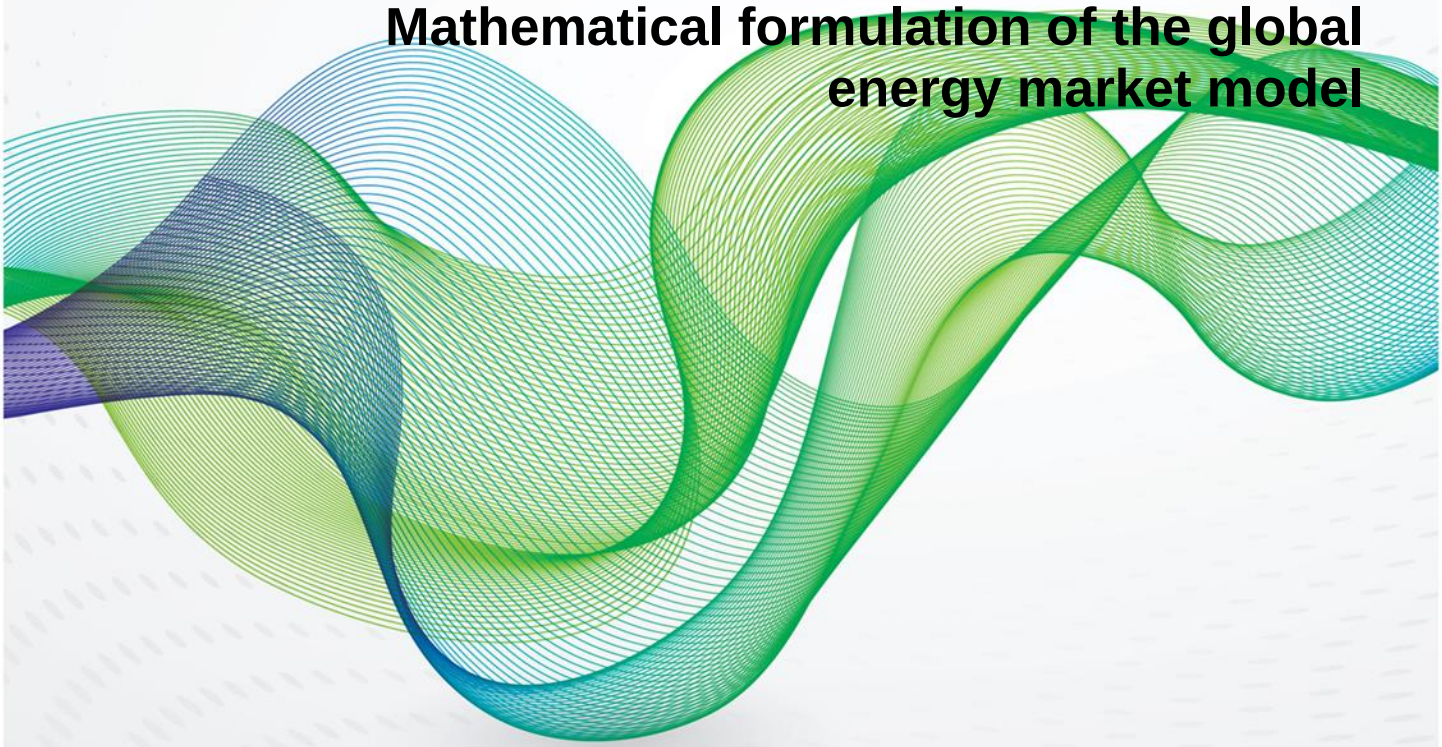
# Burning the Bridge to Ostpolitik?

# Stress-Testing Europe's Shift from Russian Gas to Renewables Using a Global Energy Model

## Supplementary Information (SI) 1: Mathematical formulation of the global energy market model



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This section documents a mathematical formulation of the global gas market model. Table A. 1 presents the nomenclature used in the formulation. The objective of this model is to minimise the total cost, which consists of variable costs of the value chain for gas and power markets subject to a set of techno-economic constraints. We use data inputs and assumptions outlined in Appendix 2 to solve this model numerically.

## A1. Notations

**Table A.1: Nomenclature**

| | **Description** | **Units** |
|---|---|---|
| **Sets** | | |
| $t \in T$ | Discrete time steps representing calendar months considered in the modelling | |
| $y \in Y$ | Discrete time steps representing calendar years considered in the modelling | |
| $n \in N$ | Set of all nodes in the model | |
| $z \in Z \subset N$ | Set of gas production nodes, a subset of all nodes N | |
| $s \in S \subset N$ | Set of gas storage nodes, a subset of all nodes N | |
| $m \in M \subset N$ | Set of gas market (end-use) nodes, a subset of all nodes N | |
| $l \in L$ | Set of steps used to linearise cost functions | |
| $j \in J$ | Set of electricity generation and storage technologies | |
| $i \in I$ | Set of all commodities considered in the model | |
| **Decision variables – gas** | | |
| $gprod_{y,t,z}$ | Gas production at time t and year y | bcm/m |
| $gflow_{y,t,n,nn}^{pipe}$ | Gas flow via pipelines at time t and year y | bcm/m |
| $gflow_{y,t,n,nn,l}^{LNG}$ | Gas flow via pipelines at time t and year y | bcm/m |
| $gstor_{y,t,s}^{IN}$ | Gas storage injection at time t and year y | bcm/m |
| $gstor_{y,t,s}^{OUT}$ | Gas storage withdrawal at time t and year y | bcm/m |
| **Auxiliary variables - gas** | | |
| $gstorlevel_{y,t,s}^{intra}$ | Gas storage level at time t | bcm |
| $gstorlevel_{y,s}^{inter}$ | Gas storage level at the beginning of year y | bcm |
| $gstorlevel_{y,t,s}^{total}$ | Total gas storage level at time t and year y | bcm |
| $gloadshed_{y,t,m}$ | Gas load shedding at node m | bcm/m |
| **Decision variables – electricity** | | |
| $eoutput_{y,tj,n}$ | Electricity generation by technology j at t and year y | $MWh_e$/m |
| $eICflow_{y,tj,n}$ | Electricity import/export (+/-) with external markets[1] | $MWh_e$/m |
| $estor_{y,t,j,n}^{IN}$ | Electricity storage injection | $MWh_e$/m |

[1] See Chyong and Newbery (2022) for details

| | | |
|---|---|---|
| $estor^{OUT}_{y,t,j,n}$ | Electricity storage withdrawal | $MWh_e$/m |
| $eflow_{y,t,n,nn}$ | Electricity flows between nodes n and nn | $MWh_e$/m |
| $ecurtail_{y,t,j,n}$ | Electricity output curtailment | $MWh_e$/m |
| $ehydrospill_{y,t,j,n}$ | Hydroelectricity output curtailment | $MWh_e$/m |
| **Auxiliary variables – electricity** | | |
| $egasdem_{y,t,n}$ | Total gas demand for electricity generation | bcm/m |
| $efueldem_{y,t,j,i,n}$ | Fuel demand for electricity generation by generator j | $MWh_{th}$/m |
| $eloadshed_{y,t,n}$ | Electricity load shedding at node n | $MWh_e$/m |
| $estorlevel^{intra}_{y,t,j,n}$ | Electricity storage level at time t | $MWh_e$ |
| $estorlevel^{inter}_{y,j,n}$ | Electricity storage level at the beginning of year y | $MWh_e$ |
| $estorlevel^{total}_{y,t,j,n}$ | Total Electricity storage level at time t and year y | $MWh_e$ |
| **Input parameters – gas** | | |
| $gDEM_{y,t,m}$ | Gas demand at node m | bcm/m |
| $\overline{gPROD_{y,z}}$ | Gas production capacity | bcm/m |
| $\overline{gFLOW^{pipe}_{y,n,nn}}$ | Gas pipeline flow capacity | bcm/m |
| $\overline{gFLOW^{LNG}_{y,n,nn}}$ | LNG flow capacity | bcm/m |
| $gLoss^{pipe}_{y,n,nn}$ | Gas pipeline losses, as a ratio of gas being sent through (n,nn) | Unitless |
| $gLoss^{LNG}_{y,n,nn}$ | LNG shipping losses, as a ratio of LNG being sent through (n,nn) | Unitless |
| $gDIST^{LNG}_{n,nn}$ | LNG shipping distance between n and nn | Nautical miles |
| $\overline{gSHIP^{LNG}_{y,t}}$ | LNG shipping capacity | bcm-nautial miles/m |
| $gSHIPTIME^{LNG}_{t,n,nn}$ | LNG shipping time | days |
| $\overline{gSHIPSTEP^{LNG}_{l}}$ | LNG shipping capacity market segments (to linearise shipping cost function) | Unitless |
| $\overline{gSHIP^{LNG-Suez}}$ | LNG shipping capacity through the Suez Canal | bcm/m |
| $\overline{gSHIP^{LNG-PM}}$ | LNG shipping capacity through the Panama Canal | bcm/m |
| $\overline{gSHIP^{LNG-NSR}}$ | LNG shipping capacity through the North Sea Route | bcm/m |
| $\overline{gLIQ^{LNG}_{y,n,nn}}$ | LNG liquefaction capacity | bcm/m |
| $\overline{gREGAS^{LNG}_{y,n,nn}}$ | LNG regasification capacity | bcm/m |
| $\overline{gSTOR^{IN}_{s,y}}$ | Storage injection capacity | bcm/m |
| $\overline{gSTOR^{OUT}_{s,y}}$ | Storage withdrawal capacity | bcm/m |
| $\overline{gSTOR^{Level}_{s,y}}$ | Storage working volume capacity | bcm |
| $\overline{gSTOR^{End}_{s}}$ | Storage level at the end of the modelling horizon | bcm |

| | | |
|---|---|---|
| $\overline{gSTOR^{Min}_{y,t,s}}$ | Storage minimum stock level | bcm |
| $\overline{gSTOR^{Max}_{y,t,s}}$ | Storage maximum stock level | bcm |
| $gSTOR^{Init}_{y,s}$ | Initial storage stock | bcm |
| $gPCOST^{A}_{z,y}$ | Gas production cost function - intercept | \$/tcm |
| $gPCOST^{B}_{z,y}$ | Gas production cost function - slope | \$/tcm$^2$ |
| $gFCOST^{Flow}_{n,nn,y}$ | Gas transport cost | \$/tcm |
| $gFCOST^{LNG}_{l}$ | LNG shipping cost | \$/day/tcm |
| $gSCOST^{A}_{y,s}$ | Gas storage cost function - intercept | \$/tcm |
| $gSCOST^{B}_{y,s}$ | Gas storage cost function - slope | \$/tcm$^2$ |
| $gDCOST^{LoadShed}_{y,m}$ | Gas load shedding cost | \$/tcm |
| **Input parameters – electricity** | | |
| $eDEM_{y,t,n}$ | Electricity demand | MWh$_e$/m |
| $eEXOG\_GEN_{y,t,j,n}$ | Exogenous electricity generation | MWh$_e$/m |
| $eGEN\_SC_{j,n}$ | Electricity self-consumption by generator j (a fraction of gross generation) | unitless |
| $\overline{eGENCAP}_{j,n,y}$ | Electricity generation capacity | MW$_e$ |
| $eHR_{j,n,y}$ | Heat rate of generator j | MW$_{th}$/MW$_e$ |
| $eCI_{j,n,y}$ | Carbon intensity of generator j | tCO$_{2e}$/MWh$_e$ |
| $\overline{eFSUPPLY}_{i,n,y}$ | Supply of commodity i for power generation | MWh$_{th}$/m |
| $\overline{eSTOR^{IN}_{j,n,y}}$ | Electricity storage charge capacity | MW$_e$ |
| $\overline{eSTOR^{OUT}_{j,n,y}}$ | Electricity storage discharge capacity | MW$_e$ |
| $\overline{eSTOR^{Level}_{j,n,y}}$ | Electricity storage working volume capacity | MWh$_e$ |
| $\overline{eSTOR^{Min}_{y,j,n}}$ | Electricity storage minimum stock level | MWh$_e$ |
| $\overline{eFLOW^{power}_{n,nn,y}}$ | Electricity cross-zonal flow limit | MWh$_e$/m |
| $eHYDRO^{INFLOW}_{y,t,j,n}$ | Exogenous hydroelectricity generation | MWh$_e$/m |
| $eSTOR^{Init}_{y,j,n}$ | Initial electricity storage stock | MWh$_e$ |
| $eFCOST^{A}_{n,i,y}$ | Fuel supply cost function - intercept | \$/MWh$_{th}$ |
| $eFCOST^{B}_{n,i,y}$ | Fuel supply cost function - slope | \$/MWh$_{th}^2$ |
| $eGCOST^{Carbon}_{n,y}$ | Fuel supply cost function - intercept | \$/tCO$_{2e}$ |
| $eGCOST^{Var}_{j,n,y}$ | Variable generation cost | \$/MWh$_e$ |
| $eDCOST^{LoadShed}_{m}$ | Electricity load-shedding cost | \$/MWh$_e$ |

## A2. Mathematical formulation

This optimisation problem minimises the total cost of meeting gas and electricity demand, consisting of (eq. A1a-i) while meeting a set of constraints (eq A2-34).

$$\sum_{y,t,z} gprod_{y,t,z}\left(gprod_{y,t,z} \times gPCOST^{B}_{z,y} + gPCOST^{A}_{z,y}\right) \quad \text{(A1a)}$$

$$\sum_{y,t,n,nn} gflow^{pipe}_{y,t,n,nn} \times gFCOST^{Flow}_{n,nn,y} \quad \text{(A1b)}$$

$$\sum_{y,t,n,nn,l} gflow^{LNG}_{y,t,n,nn,l}\left(gFCOST^{LNG}_{l} \times gSHIPTIME^{LNG}_{t,n,nn} + gFCOST^{Flow}_{n,nn,y}\right) \quad \text{(A1c)}$$

$$\sum_{y,t,s} \frac{gstor^{IN}_{y,t,s}\left(gSCOST^{B}_{y,s} \times gstor^{IN}_{y,t,s} + gSCOST^{A}_{y,s}\right)}{2} + \sum_{y,t,s} \frac{gstor^{OUT}_{y,t,s}\left(gSCOST^{B}_{y,s} \times gstor^{OUT}_{y,t,s} + gSCOST^{A}_{y,s}\right)}{2} \quad \text{(A1d)}$$

$$\sum_{y,t,m} gloadshed_{y,t,m} \times gDCOST^{LoadShed}_{y,m} \quad \text{(A1e)}$$

$$\sum_{y,t,j,i,n} efueldem_{y,t,j,i,n}\left(efueldem_{y,t,j,i,n} \times eFCOST^{B}_{n,i,y} + eFCOST^{A}_{n,i,y}\right) \quad \text{(A1f)}$$

$$\sum_{y,t,j,n} eoutput_{y,tj,n} \times eCI_{j,n,y} \times eGCOST^{Carbon}_{n,y} \quad \text{(A1g)}$$

$$\sum_{y,t,j,n} \left(eoutput_{y,tj,n} + eEXOG\_GEN_{y,t,j,n}\right) eGCOST^{Var}_{j,n,y} \quad \text{(A1h)}$$

$$\sum_{y,t,n} eloadshed_{y,t,n} \times eDCOST^{LoadShed}_{m} \quad \text{(A1i)}$$

where (1a-e) are costs related to gas supply and (1f-i) are costs related to electricity generation.

### A2.1 Gas market constraints

$$\left(gDEM_{y,t,n} - gloadshed_{y,t,n}\right) + egasdem_{y,t,n} + gstor^{IN}_{y,t,n} + \sum_{nn} gflow^{pipe}_{y,t,n,nn} + \sum_{nn,l} gflow^{LNG}_{y,t,n,nn,l} = \sum_{nn} gflow^{pipe}_{y,t,nn,n}\left(1 - gLoss^{pipe}_{y,nn,n}\right) + \sum_{nn,l} gflow^{LNG}_{y,t,nn,n,l}\left(1 - gLoss^{LNG}_{y,nn,n}\right) + gprod_{y,t,n} + gstor^{OUT}_{y,t,n}, \quad \forall y,t,n \quad \text{(A2)}$$

$$gprod_{y,t,z} \leq \overline{gPROD_{y,z}}, \quad \forall y,t,z \quad \text{(A3)}$$

$$gflow^{pipe}_{y,t,n,nn} \leq \overline{gFLOW^{pipe}_{y,n,nn}}, \quad \forall y,t,n,nn \quad \text{(A4)}$$

$$\sum_{l} gflow^{LNG}_{y,t,n,nn,l} \leq \overline{gFLOW^{LNG}_{y,n,nn}}, \quad \forall y,t,n,nn \quad \text{(A5)}$$

$$\sum_{n,nn} gflow^{LNG}_{y,t,n,nn,l} \times gDIST^{LNG}_{n,nn} \leq \overline{gSHIPSTEP^{LNG}_{l}} \times \overline{gSHIP^{LNG}_{y,t}}, \quad \forall y,t,l \quad \text{(A6)}$$

$$\sum_{n,nn,l} gflow^{LNG}_{y,t,n,nn,l} \leq gSHIP^{LNG-Suez}, \quad \forall y,t \tag{A7}$$

$$\sum_{n,nn,l} gflow^{LNG}_{y,t,n,nn,l} \leq \overline{gSHIP^{LNG-PM}}, \quad \forall y,t \tag{A8}$$

$$\sum_{n,nn,l} gflow^{LNG}_{y,t,n,nn,l} \leq \overline{gSHIP^{LNG-NSR}}, \quad \forall y,t \tag{A9}$$

$$\sum_{nn} gflow^{pipe}_{y,t,n,nn} \leq \sum_{nn} \overline{gLIQ^{LNG}_{y,n,nn}}, \quad \forall y,t,n \tag{A10}$$

$$\sum_{nn} gflow^{pipe}_{y,t,nn,n} \leq \sum_{nn} \overline{gREGAS^{LNG}_{y,nn,n}}, \quad \forall y,t,n \tag{A11}$$

$$gstor^{IN}_{y,t,s} \leq \overline{gSTOR^{IN}_{s,y}}, \quad \forall y,t,s \tag{A12}$$

$$gstor^{OUT}_{y,t,s} \leq \overline{gSTOR^{OUT}_{s,y}}, \quad \forall y,t,s \tag{A13}$$

$$gstorlevel^{total}_{y,t,s} \leq \overline{gSTOR^{Level}_{s,y}}, \quad \forall y,t,s \tag{A14}$$

$$gstorlevel^{total}_{y,t,s} \geq \overline{gSTOR^{End}_{s}}, \quad \forall y,t,s \tag{A15}$$

$$gstorlevel^{total}_{y,t,s} \geq \overline{gSTOR^{Min}_{y,t,s}}, \quad \forall y,t,s \tag{A16}$$

$$gstorlevel^{total}_{y,t,s} \leq \overline{gSTOR^{Max}_{y,t,s}}, \quad \forall y,t,s \tag{A17}$$

$$gstorlevel^{intra}_{y,t,s} = gstorlevel^{intra}_{y,t-1,s} + gstor^{IN}_{y,t,s} - gstor^{OUT}_{y,t,s}, \quad \forall y,t,s \tag{A18}$$

$$gstorlevel^{inter}_{y,s} = gSTOR^{Init}_{y,s} + gstorlevel^{inter}_{y-1,s} + gstorlevel^{intra}_{y-1,t|ord(t)=card(t),s}, \quad \forall y,s \tag{A19}$$

$$gstorlevel^{total}_{y,t,s} = gstorlevel^{inter}_{y,s} + gstorlevel^{intra}_{y,t,s}, \quad \forall y,t,s \tag{A20}$$

## A2.2 Electricity market constraints

$$\sum_{j}\left(eoutput_{y,tj,n} + eICflow_{y,tj,n}\right) \times \left(1 - eGEN\_SC_{j,n}\right) \tag{A21}$$
$$+ \sum_{j}\left(eEXOG\_GEN_{y,t,j,n} - ecurtail_{y,t,j,n}\right) + \sum_{j} estor^{OUT}_{y,t,j,n} + \sum_{nn} eflow_{y,t,nn,n}$$
$$= \left(eDEM_{y,t,n} - eloadshed_{y,t,n}\right) + \sum_{nn} eflow_{y,t,n,nn} + \sum_{j} estor^{IN}_{y,t,j,n}, \quad \forall y,t,n$$

$$eoutput_{y,tj,n} \leq \overline{eGENCAP}_{j,n,y}, \quad \forall y,t,j,n \tag{A22}$$

$$\sum_{j} eoutput_{y,tj,n} \times eHR_{j,n,y} \leq \overline{eFSUPPLY}_{i,n,y}, \quad \forall t,i,n,y \tag{A23}$$

$$estor^{IN}_{y,t,j,n} \leq \overline{eSTOR^{IN}_{j,n,y}}, \quad \forall y,t,j,n \tag{A24}$$

$$estor^{OUT}_{y,t,j,n} \leq \overline{eSTOR^{OUT}_{j,n,y}}, \quad \forall y,t,j,n \tag{A25}$$

$$estorlevel^{total}_{y,t,j,n} \leq \overline{eSTOR^{Level}_{j,n,y}}, \quad \forall y,t,j,n \tag{A26}$$

$$estorlevel^{total}_{y,t,j,n} \geq \overline{eSTOR^{Min}_{y,j,n}}, \quad \forall y,t|ord(t) = card(t),j,n \tag{A27}$$

$$eflow_{y,t,n,nn} \leq \overline{eFLOW^{power}_{n,nn,y}}, \quad \forall y,n,nn,t \tag{A28}$$

$$-\overline{eGENCAP}_{j,n,y} \leq eICflow_{y,tj,n} \leq \overline{eGENCAP}_{j,n,y}, \quad \forall y,t,j,n \tag{A29}$$

$$estorlevel^{intra}_{y,t,j,n} = estorlevel^{intra}_{y,t-1,j,n} + \left(eHYDRO^{INFLOW}_{y,t,j,n} - ehydrospill_{y,t,j,n}\right) + estor^{IN}_{y,t,j,n} \times \left(1 - eGEN\_SC_{j,n}\right) - estor^{OUT}_{y,t,j,n}, \quad \forall y,t,j,n \tag{A30}$$

$$estorlevel^{inter}_{y,j,n} = eSTOR^{Init}_{y,j,n} + estorlevel^{inter}_{y-1,j,n} + estorlevel^{intra}_{y-1,t|ord(t)=card(t),s}, \quad \forall y,j,n \tag{A31}$$

$$estorlevel^{total}_{y,t,j,n} = estorlevel^{inter}_{y,j,n} + estorlevel^{intra}_{y,t,j,n}, \quad \forall y,t,j,n \tag{A32}$$

$$egasdem_{y,t,n} = \sum_{j} eoutput_{y,tj,n} \times eHR_{j,n,y} \tag{A33}$$

$$efueldem_{y,t,j,i,n} = eoutput_{y,tj,n} \times eHR_{j,n,y} \tag{A34}$$