Emissions Trading with Profit-Neutral Permit Allocations

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Abstract

This paper examines the impact of an emissions trading scheme (ETS) on industry output, price, costs, emissions, market shares, and profits. We develop formulae for the number of emissions permits that have to be freely allocated to firms in order to neutralize any adverse impact the ETS may have on profits. Under quite general conditions, industry profits are preserved so long as firms are freely allocated a *fraction* of their total demand for permits, with this fraction lower than the industry's Herfindahl index. Our results have important implications for ETS design, especially for its ability to raise government revenue.

Keywords: Cap-and-trade, permit allocation, profit-neutrality, cost pass-through, abatement, grandfathering.

JEL classifications: D43, H23, Q58.

1 Introduction

There is increasingly broad recognition that greenhouse gas emissions are contributing to changes to Earth's climate and that reducing emissions constitutes an important challenge of economic policy (Stern, 2008). Emerging trading schemes for CO_2 and other greenhouse gases can draw upon considerable experience from other environmental markets, including the acid rain markets in sulfur dioxide (SO_2) and nitrogen oxides (NO_x) created by the 1990 Clean Air Act Amendments,[1] and other trading schemes for water and fishery rights.

The intellectual justification for economic instruments, such as emissions trading and emissions taxes, arises from the observation that, under certain assumptions, imposing a common price on emissions equalizes marginal abatement costs across the polluting firms and minimizes the aggregate cost of pollution control.[2] In most cases, this makes economic instruments more efficient than "command-and-control" intervention, which specifies input or output standards or technologies.[3] Cost efficiency is perhaps particularly important when considering policies to reduce greenhouse gas emissions, given that these are found in virtually all aspects of production and consumption. However, there is a significant disadvantage to the use of taxes or trading: while "command-and-control" intervention may impose higher marginal abatement costs, inframarginal wealth transfers, in the form of payments of taxes or for emissions permits impose an additional burden on industry and lead to political resistance. The extent to which this burden can be alleviated affects the magnitude of emissions reductions that are politically feasible.

Policy makers have sought to alleviate this problem by implementing trading schemes where all or some of the emissions permits are given for free. This is often referred to as *grandfathering* since the number of permits freely allocated to a firm is typically related to its past emissions. Grandfathering is the preferred means of winning industry support because it relieves the financial burden of the ETS on industry, without affecting firms'

[1] See Schmalensee et al. (1998) and Montero (1999).

[2] See Baumol and Oates (1988) and its references, in particular Montgomery (1972).

[3] Different standards also differ considerably in their impact and efficacy (Helfand, 1991).

incentives to reduce emissions at the margin. Indeed, the ease with which grandfathering can be coupled with an emissions trading scheme is one reason for the popularity and success of such schemes.[4,5]

For most emissions trading schemes in the US, and also in the early phases of the European Union's ETS for CO_2 (henceforth to be referred to as the EU ETS), almost all permits were freely allocated in this manner. It is clear that not selling permits (at auction, say) entails a significant loss of government revenue which could potentially be more productively employed in other ways.[6] In particular, revenue raised from the sale of permits could allow for the reduction of distortionary taxes imposed on other parts of the economy; this "revenue recycling effect" is important when evaluating the benefits of an ETS (see Fullerton and Metcalf (2001) and Bovenberg, Goulder, and Gurney (2005)). Furthermore, just as a firm's incentive to reduce emissions is unaffected by the free allocation of permits, its incentive to raise prices in response to the higher marginal cost is also unaffected by the free allocation of permits. This raises the possibility that firms will make windfall profits from free permit allocations. For these reasons and others, the question of whether to freely allocate permits, and if so, to what extent, is an important one.[7]

The aim of our paper is to provide a basic theoretical framework in which this and

[4]Another method of protecting average profits in an industry is to hold an auction for emissions permits but to then return the revenue back to the firms using some other formula. This was originally proposed by Hahn and Noll (1982); a small fraction of the permits in the Sulfur Allowance Program is allocated through a zero-revenue auction (see Tietenberg (2006, Chapter 6)).

[5]Greenhouse gas emissions trading schemes are adopted more widely than carbon taxes, even though the latter have the same cost efficiency properties as an ETS under certainty and may be even more efficient than an ETS under uncertainty (Weitzman, 1974; Pizer, 2002). See Hepburn (2006) and Stern (2008) for a discussion of the reasons behind this policy 'bias'.

[6]The allocation process can also become the focus of much rent-seeking behavior; for an account of this process in the case of the Acid Rain Program, see Joskow and Schmalensee (1998).

[7]Indeed, it is an important policy issue in existing emissions trading schemes such as the EU ETS, as well as in proposals for cap-and-trade schemes in Australia and in the US, for example, where the value of permits is likely to total around US$100 billion annually.

other central issues related to the ETS can be analyzed. We construct a model that we think serves as a very natural starting point for such an analysis; we show that, in an important set of empirically relevant cases, the number of permits required by an industry for profit-neutrality is a small fraction of its demand for permits, which means that a profit-neutral ETS can still raise substantial revenue for government through the sale of permits.

We assume that the industry is a Cournot oligopoly in its product market and a price-taker in the market for emissions permits. These are reasonable assumptions since we have in mind a scheme for trading greenhouse gas emissions, like the EU ETS, where permits are traded across many industries in (potentially) many countries while individual industries have oligopolistic structures. We show that two conditions on the model guarantee, amongst other things, that the ETS has the desired effect of reducing emissions: (i) firms' (constant) marginal costs are (weakly) positively correlated with their emissions intensities[8] and (ii) the industry faces a log-concave demand function.[9]

The imposition of a price on emissions will always encourage firms to engage in abatement, thus (weakly) lowering each firm's emissions intensity. But the ETS also leads to changes in output, so that the industry's *average* emissions intensity can increase if it is the dirtier firms that gain market share. We show that this possibility is excluded by conditions (i) and (ii). Together, these conditions ensure that firms with lower marginal costs gain market share and (ii) also guarantees that these firms are *not* more emissions intensive. In this way, we may conclude that average emissions intensity is reduced (see Proposition 6).

This mechanism also ensures that when the permit price is sufficiently low, the ETS will improve cost efficiency, i.e., the industry's average unit cost of production will *fall*.

[8]By 'emissions intensity' we mean emissions per unit of output. Note that condition (i) is consistent with notions of eco-efficiency (see Section 3 for more discussion). It is also satisfied if firms do not differ significantly in their emissions intensities.

[9]Log-concavity is a commonly-made restriction on the demand function; it is a sufficient (and, in a certain sense, necessary) condition for the Cournot oligopoly to be a game of strategic substitutes (see Section 3 for more discussion).

(We are referring to costs *excluding the cost of permits.*) At first blush, this result is surprising because the ETS causes firms to substitute away from emissions by using more of other inputs, which tends to raise costs. However, by the envelope theorem, this effect is of second order, so that the only local determinant of average unit costs is the change in market shares caused by the ETS. The ETS lowers average unit cost in the industry because it causes lower-cost firms to gain market share (see Proposition 5).

The gain in market share of lower-cost firms is one reason why the adverse profit impact (averaged across the whole industry) of the ETS tends to be limited. We measure the profit impact by looking at the *profit-neutral permit allocation*, i.e., the number of permits that have to be freely allocated to the industry to guarantee that aggregate industry profit before and after the introduction of the ETS are at the same level. Let x be the number of permits required to cover the industry's pre-ETS emissions (had the permits been needed). The profit-neutral permit allocation is always below Hx, where H is the industry's Herfindahl index (see Propositions 10 and 11).[10] For many industries, the Herfindahl index is *much* lower than 0.5. In cases like this, our result says that a large proportion of permits—perhaps more than 50%—can be auctioned whilst preserving total industry profit.

Our results on profit-neutral permit allocations are obtained by developing a first-order approach that allows us to derive simple expressions for profit-neutral permit allocations at the firm- and industry-level when the permit price is "small". These formulae, together with assumptions (i) and (ii), give us the bound on profit-neutral permit allocations outlined in the previous paragraph.[11] An attractive feature of our formulae is that they involve familiar parameters that can often be estimated with a reasonable degree of accuracy, making them amenable to empirical implementation. We illustrate this by applying them to calculate the profit-neutral permit allocation in the UK cement industry (which is included in the EU ETS). This application also shows

[10]Strictly speaking, this result holds only when the Herfindahl index is not too low in some specific sense, but the condition is likely to be satisfied in practice.

[11]We also show (in Appendix C) that our local results remain valid when the permit price is "large".

that profit-neutral permit allocations can remain partial, i.e., less than 100%, and low even if we depart significantly from assumptions (i) and (ii).

Our approach inevitably ignores other interesting issues relating to emissions trading, including some that may have an impact on profit-neutral permit allocations.[12] Chief among our assumptions is that permit allocations affect firm profits, but not firm behavior. This will be violated in situations where the market for permits is not significantly broader than the product market; Hahn (1984) and Liski and Montero (2006) consider market power in the emissions market, motivated by the markets for acid rain and particulates.[13] Moreover, we treat the number of firms in the industry as fixed; allowing for potential entry would mean that the manner in which entrants are treated by the permit allocation rules also has strategic and welfare implications. In an intertemporal setting, allocation rules may also have strategic consequences insofar as a firm's actions in one period can affect its allocation in subsequent periods. Finally, in general equilibrium models where there are pre-existing market imperfections, whether permit revenues are used to compensate firms or for some other purpose has efficiency (and not just distributional) consequences (see, e.g., Fullerton and Metcalf (2001) and Bovenberg, Goulder, and Gurney (2005)).

The rest of the paper is organized as follows. Section 2 discusses some general principles underlying profit-neutral permit allocations for general monopoly and oligopoly settings. Section 3 examines the impact of an ETS on a Cournot industry in terms of firms' outputs, market shares, costs, emissions, and the market price. Section 4 presents our results on profit-neutral permit allocations at the firm- and industry-level, and Section 5 concludes. Appendix A contains some of our proofs, Appendix B provides background on the UK cement industry, and Appendix C contains our analysis for a "large" permit price.

[12]See also Tietenberg (2006) for a careful summary of these points.

[13]Clearly, the presence of transaction costs also means that initial permit allocations have strategic consequences (Stavins, 1995), although there is some evidence of transaction costs being low in the US sulfur dioxide scheme (see Joskow, Schmalensee, and Bailey, 1998).

2 The impact of the ETS: General principles

This section spells out the major themes that are relevant for analyzing the impact of an emissions trading scheme (ETS) on firm profits. We consider an industry that produces a particular type of emission (e.g., carbon dioxide) that is harmful to the environment. The ETS imposes a cost on all emissions of this type. We assume that the industry is one of many covered by the scheme, so that, although firms have market power in their product market, they are price-takers in the permit market. In this section, we make no substantive assumptions regarding the nature of the strategic interaction in the industry.

2.1 The monopoly case

We begin by considering the case of a monopoly. Although this case is not typical, it has the merit of having a completely general solution and it provides a natural setting to introduce several of our major themes. We assume that the monopolist chooses a production plan that maximizes its profit, given the demand for its output (which may consist of one or several distinct products), its production set, input prices, and the emissions permit price $t \geq 0$. We denote the monopolist's (maximum) profit by $\Pi^*(t)$, and the associated level of emissions by $\zeta^*(t)$. Assuming that one permit is required for each unit of emissions, the profit *before* accounting for the cost of permits is $\underline{\Pi}^*(t) = \Pi^*(t) + t\zeta^*(t)$. (Note that $\Pi^*(0) = \underline{\Pi}^*(0)$.)

The situation before the introduction of the ETS corresponds to the case where $t = 0$, i.e., emissions are unpriced. Therefore, $\Pi^*(0)$ and $\zeta^*(0)$ are the monopolist's initial levels of profits and emissions respectively. To keep our notation simple, we will usually suppress the argument by writing $\Pi^*(0)$ as Π^*, and so forth.

Profit maximization by the monopolist guarantees that, at any $t > 0$,

$$\underline{\Pi}^*(t) \leq \Pi^* \text{ and } \tag{1}$$

$$\Pi^*(t) = \underline{\Pi}^*(t) - t\zeta^*(t) \geq \Pi^* - t\zeta^*. \tag{2}$$

Note that (1) follows from the fact that Π^* is the optimal profit at $t = 0$, while the production decision that generates a profit of $\underline{\Pi}^*(t)$ is one that the monopolist *could* have made at $t = 0$, so the latter must be smaller than the former. The right-hand side of (2) is the monopolist's profit if it chooses not to adjust production after the introduction of the ETS—this must be less than $\Pi^*(t)$, which is the *optimal* profit when emissions are priced at t.

Combining (1) and (2) yields two conclusions. First, the introduction of the ETS reduces emissions, since these inequalities only hold simultaneously if $\zeta^*(t) \leq \zeta^*$. Second, the ETS reduces the monopolist's profit, since (1) implies that $\Pi^*(t) = \underline{\Pi}^*(t) - t\zeta^*(t) \leq \Pi^*$.

Consider now the level of free allocation of permits required to compensate the monopolist for the reduction in profits from Π^* to $\Pi^*(t)$. From (2), $\Pi^*(t) + t\zeta^* \geq \Pi^*$, so there is a $0 \leq \gamma(t) \leq 1$ such that

$$\Pi^*(t) + t\,[\gamma(t)\zeta^*] = \Pi^*. \tag{3}$$

In other words, $\gamma(t)\zeta^*$ is the number of freely allocated permits—the *profit-neutral allocation* (PNA)—that will leave the monopolist's total profits at the pre-ETS level. Since $\gamma(t) \leq 1$ free permits cover only a *fraction* of the firm's initial emissions. In this case, we say that the profit-neutral allocation is *partial.*

The intuition for this result is as follows. Suppose that the introduction of the ETS is accompanied by a free allocation of permits at the monopolist's original level of emissions; furthermore, suppose that the monopolist chooses *not* to adjust its production plan in response to the introduction of the ETS. Then the increase in her costs would be *exactly* offset by the value of the free allowances. However, the *option* to adjust (e.g., increase price(s) or switch to cleaner inputs) means that the PNA, in general, is partial. It is worth emphasizing that this conclusion is very robust: no restrictions are imposed on the monopolist, except that it is a price-taker in the market for emissions permits.[14]

[14]Furthermore, it is clear that the result holds even if the monopolist is subject to certain regulatory restrictions, such as being prevented from raising prices after the introduction of the ETS.

Finally, suppose that the monopolist indeed receives the PNA of $\gamma(t)\zeta^*$ permits for free. Re-writing (3), we obtain that $\underline{\Pi}^*(t) + t\,[\gamma(t)\zeta^* - \zeta^*(t)] = \Pi^*$. This, together with (1), implies that the monopolist's endowed permits under the PNA, $\gamma(t)\zeta^*$, will *exceed* its requirement $\zeta^*(t)$, so the monopolist will be selling part of its endowment.

The following proposition summarizes our analysis of the monopoly case.

PROPOSITION 1 *Following the introduction of the ETS, a monopolist has lower emissions and lower profit. PNA is partial, i.e.,* $0 \leq \gamma(t) \leq 1$*; with this allocation of permits the monopolist is a net supplier in the market for permits.*

2.2 Characterizing partial PNA

When considering an oligopoly, we can no longer rely solely on the revealed preference arguments that gave us such mileage in the monopoly case. Nevertheless, there are still some general insights we can derive.

Assume that there are $N \geq 2$ firms in an industry that interact with each other strategically. In the interest of generality, we leave the precise manner of their strategic interaction unspecified for now. Retaining our earlier notation, we denote equilibrium industry profits when the permit price is t by $\Pi^*(t)$, the equilibrium (total) emissions by $\zeta^*(t)$, and so on. The corresponding outcomes for firm i are $\Pi_i^*(t)$, $\zeta_i^*(t)$, etc. We assume that these are all smooth functions of the permit price t in some interval $[0, T]$, where $T > 0$. We call this model a *smooth oligopoly.*

By definition, the proportion of free permit allocation needed for profit-neutrality at the industry-level, $\gamma(t)$, is given by

$$\Pi^*(t) + t\,\gamma(t)\zeta^* = \Pi^*. \tag{4}$$

The next result gives a sufficient (and, as we shall see later on, locally necessary) condition for the profit-neutral allocation to be partial.

PROPOSITION 2 *Suppose* $\zeta^*(t) < \zeta^*$ *and* $\underline{\Pi}^*(t) \geq \Pi^*$*. Then* $\gamma(t) < 1$*; with this level of free allocation, the industry has a net demand for permits.*

Proof: Given the assumptions, there is a $\gamma(t) < 1$ such that $\underline{\Pi}^*(t) - \Pi^* + t\left[\gamma(t)\zeta^* - \zeta^*(t)\right] = 0$. Rearranging this expression and using the fact that $\underline{\Pi}^*(t) - t\zeta^*(t) = \Pi^*(t)$, we obtain (4). Since $\underline{\Pi}^*(t) \geq \Pi^*$, we must have $\gamma(t)\zeta^* - \zeta^*(t) \leq 0$, so the industry has a net demand for permits. QED

This result is quite intuitive. It says that the industry PNA is partial if the introduction of an ETS increases industry profits *before* accounting for emissions costs—in other words, *average PNA is partial if the ETS leads to a more "collusive" equilibrium outcome.* In particular, had the firms in the industry chosen the (same) actions they did upon the introduction of the ETS *before* it was introduced, their total profits (at $\underline{\Pi}^*(t)$) would have exceeded Π^*.[15]

To carry the analysis further, we now concentrate on the behavior of $\gamma(t)$ for low values of t by examining the first-order approximation $\tilde{\gamma} \equiv \lim_{t\to 0} \gamma(t)$. This limit determines the (approximate) proportion of free permit allocation that satisfies (4) by ignoring higher-order terms in t, and allows us to deliver sharp and easily interpretable results. The marginal cost increase due to an ETS is typically small relative to a firm's total marginal costs, so an analysis based on this approach delivers insight without being misleading.[16] We make three important observations regarding $\tilde{\gamma}$.

(1) If $\tilde{\gamma} < 1$, then for small t, $\gamma(t) < 1$, that is, the industry (on average) requires only partial PNA for profit-neutrality. Moreover, the industry's net demand for permits, assuming it is given this level of free allocation, is $\zeta^*(t) - \gamma(t)\zeta^*$. Since $\lim_{t\to 0} \zeta^*(t) = \zeta^*$, for low values of t, $\zeta^*(t) - \gamma(t)\zeta^* > 0$. In other words, *if* $\tilde{\gamma} < 1$, *the industry's demand for permits under the PNA will exceed its free allocation.* Conversely, if $\tilde{\gamma} > 1$, then the industry will be a net supplier of permits.

Suppose now that there are sufficiently many industries with partial PNA so that (overall) there is a net demand for permits after profit-neutral permit allocations. Then

[15] For example, in the standard textbook case of symmetric Cournot oligopoly with constant marginal cost, industry profits are lower than for a monopolist. If the ETS leads to a lower industry output that is closer to the monopoly level, then PNA is partial.

[16] In any case, we show in Appendix C that our main insights remain valid even when t is "large".

a given permit price of $t > 0$ can only be supported if there is an external party—the government—that meets this net demand for permits. Therefore an ETS with profit-neutral permit allocations raises net revenue for government if it is partial (on average across the industries covered by the ETS).

(2) Recall that if the industry is run by a monopoly then, for all values of t, PNA is partial, i.e., $\gamma(t) \leq 1$, but the monopoly is also a net supplier of permits. Comparing this with our previous observation, we conclude that, *for a monopoly,* $\tilde{\gamma} = 1$; so even though PNA is partial for a monopoly it approaches a full allocation of permits for low permit prices.

(3) Taking the Taylor expansion of $\Pi^*(t)$ around $t = 0$, (4) gives us a simple expression for PNA, namely

$$\tilde{\gamma} \equiv \lim_{t \to 0} \gamma(t) = -\frac{1}{\zeta^*} \frac{d\Pi^*}{dt}(0). \tag{5}$$

To first order, the proportion of free permits required for profit-neutrality is equal to the loss in industry profits per unit of emissions.[17] Since, by definition, $\Pi^*(t) = \underline{\Pi}^*(t) - \zeta^*(t)t$, we can also write

$$\tilde{\gamma} = 1 - \frac{1}{\zeta^*} \frac{d\underline{\Pi}^*}{dt}(0), \tag{6}$$

from which the next proposition follows immediately.

Proposition 3 *In a smooth oligopoly,*

$$\tilde{\gamma} < 1 \iff \frac{d\underline{\Pi}^*}{dt}(0) > 0. \tag{7}$$

This result is the local analog of Proposition 2. Indeed, it goes a bit further since it says that, for small t, the condition that $\underline{\Pi}^*(t)$ is increasing with t is both sufficient—and necessary—for partial PNA at the industry level.

[17] An alternative way of showing that $\tilde{\gamma} = 1$ for a monopolist is to first observe that, by the envelope theorem, $d\Pi^*/dt = -\zeta^*$ and then to apply formula (5).

2.3 The impact of the ETS on costs

We now consider the impact of the ETS on firm costs, taking into account both the direct effect of the permit price on costs as well as firms' abatement decisions. From this point on, we assume that the industry produces a *single* product using l (costly) inputs, represented by a vector $\bar{x}_i = (\bar{x}_i^1, \bar{x}_i^2, ..., \bar{x}_i^l)$ in R_{++}^l. Production leads to emissions, which we denote by $\bar{z}_i$. Following Baumol and Oates (1988), amongst others, we shall think of emissions as an input in the production process, albeit one that is initially free. Firm i's production function F_i, assumed to exhibit constant returns to scale, maps the input vector $(\bar{x}_i, \bar{z}_i)$ to the output q_i. All inputs (including emissions) are chosen optimally by firms to minimize costs.

The introduction of an ETS typically induces firms to engage in abatement by reducing their emissions and using more of other inputs (whose prices we assume are unchanged). We denote firm i's unit cost at permit price t by $c_i(t)$, its optimal emissions intensity (i.e., emissions per unit of output) by $z_i(t)$, and its unit cost excluding the cost of permits by $\underline{c}_i(t) \equiv c_i(t) - tz_i(t)$.

Standard production theory tells us that, at any $t > 0$, $z_i(t) \leq z_i(0)$ and $a_i(t) \equiv \underline{c}_i(t) - c_i(0) \geq 0$. We can think of $a_i(t)$ as the *abatement cost* (per unit of output) incurred by the firm as it reduces emissions intensity from $z_i(0)$ to $z_i(t)$. If the production technology is such that abatement is either non-optimal or simply impossible, then $\underline{c}_i(t) \equiv c_i(0)$. By the envelope theorem,

$$\frac{dc_i}{dt}(t) = z_i(t), \tag{8}$$

which leads to the important observation that

$$\frac{da_i}{dt}(0) = \frac{d\underline{c}_i}{dt}(0) = 0. \tag{9}$$

So even though the ETS leads to increased expenditure on other inputs, this effect is of second order.[18]

[18] A less general but more concrete way of obtaining the same conclusion is to think of firm i having

Suppose $q_i^*(t)$ is the firm's equilibrium output as a function of the permit price. Then its total cost excluding the cost of permits is $\underline{C}_i^*(t) = \underline{c}_i(t) q_i^*(t)$. Differentiating by t and appealing to (9), we obtain

$$\frac{d\underline{C}_i^*}{dt}(0) = c_i(0) \frac{dq_i^*}{dt}(0). \tag{10}$$

The introduction of the ETS thus has a two-fold impact on $\underline{C}_i^*$. First, it makes the firm switch away from emissions towards other inputs, thus raising expenditure ($\underline{c}_i(t)$) on those inputs. Second, it has an impact on firm i's output via its strategic interaction with other firms. However, by the envelope theorem, the change in $\underline{c}_i(t)$ is of second order, so that, for low values of t, the change in total cost $\underline{C}_i^*(t)$ is simply driven by the change in firm i's output.

An immediate consequence is that the local impact of an ETS on an industry's average cost is *solely* driven by its effect on firms' relative output shares. We denote the industry's total cost, again excluding permits, by $\underline{C}^*(t) = \sum_{i=1}^N \underline{C}_i^*(t)$ and the associated average cost by $\underline{c}^*(t) \equiv \underline{C}^*(t)/Q^*(t)$. Clearly,

$$\underline{c}^*(t) = \sum_{i=1}^N \sigma_i(t) \underline{c}_i(t), \tag{11}$$

where $\sigma_i(t)$ is firm i's market share. Differentiating this expression with respect to the permit price and using (9), we obtain (note that $\underline{c}_i(0) = c_i(0)$)

$$\frac{d\underline{c}^*}{dt}(0) = \sum_{i=1}^N c_i(0) \frac{d\sigma_i}{dt}(0). \tag{12}$$

It is clear from this equation that the industry's average cost (excluding permits) will *fall* with the introduction of an ETS if firms with lower unit costs increase their market share. This observation will be important for determining the average profit impact of an ETS, and therefore also the industry's PNA.

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access to a technology that reduces emissions intensity by α at a cost of $d_i(\alpha)$ per unit of output. In this case, $c_i(t) = \min_{\alpha \geq 0}[c_i(0) + (z_i(0) - \alpha)t + d_i(\alpha)]$. Denoting the cost-minimizing value of α by $\bar{\alpha}_i(t)$, we have $z_i(t) = z_i(0) - \bar{\alpha}_i(t)$ and $a_i(t) = d_i(\bar{\alpha}_i(t))$. It is easy to verify (8) and (9) directly.

3 The ETS in a Cournot model

Consider a standard Cournot oligopoly with $N \geq 2$ quantity-setting firms. Without loss of generality, assume that $c_1(0) \leq c_2(0) \leq ... \leq c_N(0)$, so lower indexed firms have lower initial marginal costs. We denote by q the vector $(q_i)_{1 \leq i \leq N}$ which gives the output of each firm, the aggregate output associated with q by Q, and the output of all firms except firm i by Q_{-i}. The marginal revenue of firm i at q satisfies $MR_i(q) = P(Q) + q_i P'(Q)$, where $P(Q)$ is the downward-sloping inverse demand curve. Firm i maximizes its profit when marginal revenue equals marginal cost, $MR_i(q) = c_i(t)$.

Before the introduction of the ETS firms are at the Cournot equilibrium $q^* = (q_i^*)_{1 \leq i \leq N}$ so that total output $Q^* = \sum_{i=1}^N q_i^*$. At this equilibrium, $MR_i(q^*) = c_i(0)$, which may be re-written as

$$c_i(0) = P(Q^*) \left[1 - \frac{\sigma_i}{\eta(Q^*)} \right] \tag{13}$$

where σ_i is firm i's initial (pre-ETS) market share and $\eta(Q^*) = |P(Q^*)/Q^* P'(Q^*)|$ is the industry price elasticity of demand. It follows from (13) that $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_N$, i.e., market shares vary inversely with marginal cost.

Let $E(Q^*) = - \left[d \log P'(Q) / d \log Q \right]_{Q=Q^*}$ denote the elasticity of the *slope* of inverse demand, evaluated at the initial equilibrium industry output. This can be interpreted as an index of demand curvature. Clearly, $E(Q^*) > 0$ ($E(Q^*) < 0$) if $P''(Q^*) > 0$ ($P''(Q^*) < 0$) and inverse demand is locally convex (concave) at Q^*. The second-order condition for profit-maximization is satisfied for firm i if its marginal revenue is downward-sloping in its own output, i.e.,

$$\frac{\partial MR_i}{\partial q_i}(q^*) = 2P'(Q^*) + q_i^* P''(Q^*) < 0; \tag{14}$$

this condition may be equivalently written as

$$2 - \sigma_i E(Q^*) > 0. \tag{15}$$

Besides (15), the other assumption maintained throughout the paper is that inverse demand is not too convex in the sense that

$$N + 1 - E(Q^*) > 0. \tag{16}$$

As we shall see, this assumption is necessary and sufficient for industry output to fall when emissions trading is introduced.[19]

Conditions (15) and (16) are not enough to guarantee that the Cournot model is well-behaved; in particular, even with these assumptions, it is possible for a firm to have an upward-sloping best response curve. It is known that the necessary and sufficient condition for the best response curve of firm i to be locally downward sloping at i's equilibrium output is

$$1 - \sigma_i E(Q^*) > 0 \tag{17}$$

(see, for example, Bulow, Geanakoplos, and Klemperer (1985) and Shapiro (1989)).[20] If this conditions holds for each firm i, then the Cournot oligopoly is locally a game of strategic substitutes. Obviously, condition (17) is stronger than (15).

A sufficient condition for (17) is that $E(Q^*) \leq 1$, which is equivalent to saying that the demand function, i.e., the function P^{-1}, is locally log-concave at $P(Q^*)$. Indeed this condition is *necessary* if (17) is to hold for any distribution of market shares. We also know that if the demand function is globally log-concave, then the Cournot game has a globally unique and stable equilibrium (Shapiro (1989)). For these reasons and others, the log-concavity of the industry demand curve is a reasonable and commonly-made assumption in the context of the Cournot model (see, for example, Farrell and Shapiro (1990) and Shapiro (1989)).[21] This assumption will also feature prominently in some of our main results.

[19] See Bergstrom and Varian (1985) for another use of this condition, and Seade (1980) for an early application that notes the importance of demand curvature.

[20] Using equation (48), it is straightforward to show that the slope of i's best response curve, $\partial \hat{q}_i / \partial Q_{-i} = -[1 - \sigma_i E(Q^*)]/[2 - \sigma_i E(Q^*)]$. Given that the denominator of this expression is positive (by (15)), the expression is negative if and only if its numerator is positive, hence condition (17).

[21] For products that are consumed only as a single unit or none at all, market demand at price p is proportional to $\bar{F}(p) = 1 - F(p)$, where F is the distribution of the reservation prices. A sufficient condition for $\bar{F}$ to be log-concave is for F to be generated by a log-concave density function. Many

The introduction of the ETS raises the marginal cost of firms in the industry. The envelope theorem tells us that $dc_i/dt = z_i(t)$ (see (8)), so the local impact of the ETS on a firm's marginal cost depends (only) on its emissions intensity. In principle, $z_i > 0$ may vary across firms in any possible way, but for some of our results we shall assume that *emission intensities are positively correlated with marginal costs.* By this we mean that their covariance is nonnegative at $t = 0$; formally,

$$\text{cov}(c, z) \equiv \frac{1}{N}\sum_{i=1}^{N} c_i z_i - \frac{1}{N^2}\sum_{i=1}^{N} c_i \sum_{i=1}^{N} z_i \geq 0. \tag{18}$$

It almost goes without saying that this condition will not always hold, but it is sufficiently weak to cover a broad range of cases. By definition, a firm with the lower marginal cost is the one that uses fewer inputs on average (with inputs weighted by their prices); that such a firm will typically also use less of the inputs that cause emissions seems plausible. Indeed, the general notion of eco-efficiency holds that reducing waste also reduces costs (Alexander and Buchholz, 1978; Porter and van der Linde, 1995), suggesting that it is plausible that costs and emissions are non-negatively correlated.[22] More recently, Bloom, Genakos, Martin and Sadun (2008) find strong empirical evidence that more efficient manufacturing firms also tend to have lower energy and emissions intensities. However, the condition is less likely to hold, for example, in an electricity market where some firms operate coal-fired power plants (with low marginal costs, but high emissions intensities), while, *in addition*, there are other firms that instead use cleaner, but also more expensive inputs (such as gas). Nevertheless, the condition is very useful for deriving a clean set of theoretical results, and the broad thrust of our conclusions—especially our main results on partial PNA—remains valid with modest departures from

commonly used density functions have this property, see Bagnoli and Bergstrom (2005).

[22]See also Heal (2008) for several case studies, such as the internal emissions trading scheme set up by BP, which reduced emissions and also cut costs; Dow Chemicals and Du Pont provide similar evidence. King and Lenox (2001), amongst others, find a positive correlation between environmental and financial performance. Although there is considerable debate on the reasons for this relationship (Konar and Cohen, 2001), for our purposes the nature and direction of causality between environmental and financial performance is irrelevant, and it suffices that they are non-negatively correlated.

this assumption (see Sections 4.2 and 4.3).

We say that emissions intensities are *co-monotonic with marginal costs* if, at $t = 0$, z_i is weakly increasing with c_i and that it is *uniform* if z_i is equal for all firms. Since marginal cost is increasing with i by assumption, it is clear that co-monotonicity (and, as a special case, uniformity) is stronger than—in fact, considerably stronger than—condition (18).

Using (13) it is straightforward to check that marginal costs and emissions intensities are positively correlated if and only if market shares and emissions intensities are negatively correlated. In other words, (18) is equivalent to

$$\operatorname{cov}(\sigma, z) = \frac{1}{N}\sum_{i=1}^{N} \sigma_i z_i - \frac{1}{N^2}\sum_{i=1}^{N} z_i \leq 0, \tag{19}$$

(note that $\sum_{i=1}^{N} \sigma_i = 1$).

For the rest of this section, we shall examine the impact of the ETS on output, market shares, emissions, and costs. Building on this, we examine the impact of the ETS on profits, and thus PNA, in Section 4.

3.1 The impact of the ETS on output and price

We assume that the Cournot equilibrium varies smoothly with t when the ETS is introduced, i.e., the Cournot oligopoly is a smooth oligopoly in the sense defined in Section 2. The following result shows the impact of the ETS on firm- and industry-level output and is crucial to understanding its impact on costs and firm profits.

Proposition 4 *Output responses at $t = 0$ are given by*

$$\frac{dQ^*}{dt} = \frac{\sum_{i=1}^{N} z_i}{P'(Q^*)\left[N + 1 - E(Q^*)\right]} < 0; \tag{20}$$

$$\frac{dQ^*_{-i}}{dt} = \frac{dQ^*}{dt}\left[\left[2 - \sigma_i E(Q^*)\right] - \left[N + 1 - E(Q^*)\right]\frac{z_i}{\sum_{j=1}^{N} z_j}\right]; \tag{21}$$

$$\frac{dq^*_i}{dt} = \frac{dQ^*}{dt}\left[-\left[1 - \sigma_i E(Q^*)\right] + \left[N + 1 - E(Q^*)\right]\frac{z_i}{\sum_{j=1}^{N} z_j}\right]. \tag{22}$$

The proofs of Proposition 4 and other results in this section and the next are in Appendix A.

Proposition 4 says that the introduction of the ETS causes industry output to fall in response to firms' increased marginal costs. It follows that the equilibrium price of output must increase. To be specific, (20) gives us

$$\frac{dP^*}{dt} = P'(Q^*)\frac{dQ^*}{dt} = \frac{\sum_{i=1}^{N} z_i}{[N+1-E(Q^*)]}. \tag{23}$$

This formula is remarkably simple in one respect: the price increase depends on only the unweighted average of the emissions intensities and on no other feature of its distribution. So a change in emissions intensities that leaves its unweighted average unchanged does not modify the price impact of the ETS.

To have a better understanding of (23), consider the hypothetical situation where $z_i = 1$ for all i. Then

$$\frac{dP^*}{dt} = \frac{N}{[N+1-E(Q^*)]} \equiv \kappa. \tag{24}$$

The term κ is known as the *rate of cost pass-through* since it measures the change in the equilibrium price following a common increase in the marginal cost of every firm in the oligopoly. Loosely speaking, if marginal cost increases by a dollar at every firm, then the equilibrium price rises by κ dollars. Note that *the rate of cost pass-through* $\kappa \leq 1$ *if and only if* $E(Q^*) \leq 1$, *i.e., demand is locally log-concave.*

Denoting the unweighted average marginal cost in the industry by $\hat{c}(t) = (\sum_{i=1}^{N} c_i(t))/N$, it follows from (8) that

$$\frac{d\hat{c}}{dt}(0) = \frac{\sum_{i=1}^{N} z_i}{N}, \tag{25}$$

so (23) may be rewritten as $dP^*/dt = \kappa\ (d\hat{c}/dt)$ .In finite terms, $\Delta P^* \approx \kappa\, \Delta\hat{c}$, so the price increase following the introduction of the ETS is approximately proportional to the rise in the unweighted marginal cost, with the cost pass-through κ as the proportionality constant.

3.2 The impact of the ETS on market shares

It is clear from Proposition 4 that the impact of the ETS does not fall equally across firms. However, while the scheme will affect relative output and hence market shares, its effect is not indeterminate; we show that under reasonable assumptions, the introduction of the ETS will increase market concentration.

To obtain a quick sense of why this may be so, use (20) and (22) to obtain

$$\frac{d\sigma_i}{dt}(0) = \frac{1}{Q^*(0)}\frac{dQ^*}{dt}\left[-1 + \sigma_i\left[E(Q^*) - 1\right] + \left[N + 1 - E(Q^*)\right]\frac{z_i}{\sum_{j=1}^N z_j}\right]. \tag{26}$$

We wish to highlight two factors that influence the impact of the ETS on market shares: the curvature of demand and the distribution of emissions intensities. To focus on the former, assume that emissions intensities are uniform across firms, so that all firms experience the same increase in marginal cost. In this case, it is clear from (26) that $d\sigma_i/dt$ decreases with i if and only if $E(Q^*) \leq 1$. Bear in mind that $\sum_{i=1}^N d\sigma_i/dt = 0$; so if $d\sigma_i/dt$ decreases with i then $\{d\sigma_i/dt\}_{1\leq i\leq N}$ will have the *single crossing property*, i.e., there is k such that for all $i \leq k$, $d\sigma_i/dt \geq 0$ and for all $i > k$, $d\sigma_i/dt < 0$. Since lower indexed firms are also the larger ones, we see that the ETS raises the market share of large firms and reduces that of small firms when demand is log-concave and emissions intensities are uniform.

There is a different and perhaps more intuitive way of understanding the influence of the demand curvature on market shares. At the Cournot equilibrium, firm i's first order condition may be written as

$$P(Q^*(t)) - c_i(t) = \left[-P'^*(t))Q^*(t)\right]\sigma_i(t)\,. \tag{27}$$

This equation says that firm i produces up to the point at which the margin gained from selling one more unit equals firm i's market share, $\sigma_i(t)$, of the revenue lost by all firms on the inframarginal units, which is $-P'^*(t))Q^*(t)$. It is straightforward to check that $L(Q) = -P'(Q)Q$ *is decreasing in* Q *if and only if* $E(Q) \leq 1$. Now consider two firms i and j, with i having the lower marginal cost (and hence larger market share); then

$$c_j(t) - c_i(t) = L(Q^*(t))[\sigma_i(t) - \sigma_j(t)]. \tag{28}$$

The introduction of the ETS lowers output and hence L, but the difference in margins between the two firms (which is $c_j(t) - c_i(t)$) remains unchanged if they have the same emissions intensity. It follows that $\sigma_i(t) - \sigma_j(t)$ must increase.

The other factor influencing the impact of the ETS on market shares is the distribution of emissions intensities. Consider firms m and n with the same (initial) market share but different emissions intensities, with $z_m < z_n$; then (26) tells us that $d\sigma_m/dt > d\sigma_n/dt$, i.e., in terms of market share, firm n will fare less well under the ETS. This is unsurprising since firm n has the greater emissions intensity and so experiences a larger increase in marginal cost than firm m.

When $E(Q^*) \leq 1$ and emissions intensities are co-monotonic with marginal costs (and hence weakly increasing in i), both factors work in favor of the larger firms; this is reflected in (26) where $d\sigma_i/dt$ decreases with i and so $\{d\sigma_i/dt\}_{1\leq i\leq N}$ has the single crossing property. It is straightforward to check that the single crossing property, together with the fact that σ_i is decreasing in i, will raise market concentration as measured by the Herfindahl index $H = \sum_{i=1}^N \sigma_i^2$; formally,

$$\frac{dH}{dt}(0) = 2\sum_{i=1}^{N} \sigma_i \frac{d\sigma_i}{dt}(0) \geq 0. \tag{29}$$

When co-monotonicity is replaced with the weaker assumption that emissions intensities and marginal costs are positively correlated, it is no longer true that $\{d\sigma_i/dt\}_{1\leq i\leq N}$ must obey the single crossing property. However, one can still show (see Appendix A) that the introduction of the ETS raises the Herfindahl index.

PROPOSITION 5 *Suppose $E(Q^*) \leq 1$ and that emissions intensity and marginal cost are positively correlated. Then*

$$\frac{dH}{dt}(0) \geq 0.$$

3.3 The impact of the ETS on average cost

The standard justification for the use of an ETS is the cost minimization theorem (see Baumol and Oates (1988)), which says that it represents the cheapest way of achieving an emissions target subject to a given set of output constraints. We now give a formal statement of this result in the context of a Cournot model.

Let r denote the input price vector of all inputs except emissions and let t be the unit price of emissions. Keeping r fixed, we denote firm i's optimal input vector (for producing a single unit of output) by $(x_i(t), z_i(t))$, where $x_i(t)$ is the vector of inputs excluding emissions and $z_i(t)$ is the emissions intensity. Thus, the unit cost excluding the cost of permits is $\underline{c}_i(t) = r \cdot x_i(t)$. As above, firm i's output at the Cournot equilibrium is $q_i^*(t)$. The problem of minimizing costs subject to achieving an emissions target can be stated as:

$$\min r \cdot \left[\sum_{i=1}^{N} \hat{x}_i \right]$$

$$\begin{aligned} \text{subject to } (i)\ F_i(\hat{x}_i, \hat{z}_i) &= q_i^*(t) \text{ for all } i, \\ \text{and } (ii) \sum_{i=1}^{N} \hat{z}_i &= \sum_{i=1}^{N} z_i(t) q_i^*(t), \end{aligned}$$

where F_i is firm i's production function, so $F_i(\hat{x}_i, \hat{z}_i)$ is the firm's output at the input vector $(\hat{x}_i, \hat{z}_i)$. This optimization determines the cheapest way of achieving an emissions target of $\sum_{i=1}^{N} z_i(t) q_i^*(t)$ subject to firm i producing $q_i^*(t)$. By the cost minimization theorem (see Baumol and Oates (1988)), the solution to this problem is $\hat{x}_i = x_i(t) q_i^*(t)$ and $\hat{z}_i = z_i(t) q_i^*(t)$. In short, the solution coincides with that achieved by an ETS with permits priced at t.

This result is difficult to interpret in a Cournot setting: since firms' outputs are perfect substitutes, there is no normative reason for imposing condition (i). However, if we replace it with the condition that *total* output equals $\sum_{i=1}^{N} q_i^*(t)$, then it is clear that neither the initial Cournot equilibrium nor that after the introduction of the ETS are cost efficient. Cost minimization in both instances would require all output to be produced by the firm with lowest marginal cost.

A more useful criterion in evaluating the impact of the ETS in this setting is to ask whether it makes the industry more (or less) efficient in its use of resources (other than emissions). Formally, we wish to study the impact of emissions trading on the industry's average cost, $\underline{c}^*(t)$. Recall that, by the envelope theorem, $d\underline{c}_i/dt = 0$, so that the local impact of the ETS on the industry's average cost is *solely* driven by its impact on relative output shares (see (9) and (12)). Therefore, cost efficiency improves if firms with lower unit costs (lower i) increase their market share. Note that the firms with lower unit costs are also the larger firms, so it should come as no surprise if the conditions in Proposition 5 that guarantee increased market concentration are also sufficient to guarantee an improvement in cost efficiency.

PROPOSITION 6 *Suppose that $E(Q^*) \leq 1$ and that emissions intensity and marginal cost are positively correlated. Then the industry's average cost obeys*

$$\frac{d\underline{c}^*}{dt}(0) \leq 0.$$

We conclude that, when t is small, it is reasonable to expect the introduction of the ETS to lead to a *reduction* in the industry's average cost of production. Since Proposition 6 relies on the fact that firm-level cost increases are of second order, it is possible that these increases could be significant enough to raise average costs when t is large. Nonetheless, even for large t, the assumptions of Proposition 6 guarantee that it is the relatively low cost firms that gain market share, which helps moderate the impact of any increase in costs.

3.4 The impact of the ETS on emissions

We already know that the ETS lowers total industry output, so total emissions will fall if the average emissions intensity of firms falls. Letting $z^*(t)$ denote average emissions intensity and noting that $z^*(t) = \sum_{i=1}^{N} \sigma(t) z_i(t)$, we obtain

$$\frac{dz^*}{dt}(0) = \sum_{i=1}^{N} z_i(0) \frac{d\sigma_i}{dt}(0) + \sum_{i=1}^{N} \frac{dz_i}{dt}(0)\sigma_i(0). \tag{30}$$

Standard production theory tells us that $dz_i/dt \leq 0$ (in other words, firms make abatement decisions), so the second term on the right of this equation is always negative. If emissions intensity is uniform across firms, the first term on the right equals zero since $\sum_{i=1}^{N} d\sigma_i/dt = 0$ and we conclude that average emissions intensity must fall.

If emissions intensity is *not* uniform, the sign of the first term on the right of (30)—and thus the sign of dz^*/dt—cannot be guaranteed without further assumptions. This reflects the fact that while the ETS induces each firm to lower its emissions intensity, it is possible for this effect to be negated in part or in whole by strategic effects. If the ETS causes firms with (initially) low emissions to gain market share, then it has a doubly beneficial effect. On the other hand, if these firms lose market share, this diminishes the scheme's ability to lower emissions in this industry.[23] To guarantee that the former holds, we once again rely on the assumptions that $E(Q^*) \leq 1$ and that emissions intensities are positively correlated with marginal costs. We know from Proposition 5 that the ETS raises market concentration under these conditions; loosely speaking, large firms gain market share at the expense of small firms. Since large firms have lower cost and (by the correlation assumption again) typically lower emissions, average emissions intensity will fall.

Proposition 7 *Average emissions intensity z^* and total emissions ζ^* satisfy*

$$\frac{dz^*}{dt}(0) \leq 0 \textit{ and } \frac{d\zeta^*}{dt}(0) \leq 0$$

if either (a) *emissions intensities are uniform or* (b) $E(Q^*) \leq 1$ *and emissions intensities are positively correlated with marginal cost.*

4 Profit-neutral permit allocations

Having established the impact of the ETS on output, price, costs and emissions, we now turn to examine its impact on profits. In particular, we develop formulae that determine the level of free permit allocations required to ensure profit-neutrality at the level of

[23] The possibility of such perverse effects has also been noted in Levin's (1985) study of taxation in a Cournot model.

the firm and of the industry. We use these formulae to show that, under reasonable conditions, average PNA in the industry is not just partial, but *low*. We also perform some illustrative calculations on the level of PNA for the UK cement industry.

4.1 PNA for an individual firm

By definition, the proportion of free permit allocation, $\gamma_i(t)$, needed to preserve firm i's profit satisfies

$$\Pi_i^*(t) + t\,\gamma_i(t)\zeta_i^* = \Pi_i^*. \tag{31}$$

Taking the Taylor expansion of $\Pi_i^*(t)$ at $t = 0$ yields the approximate PNA

$$\tilde{\gamma}_i \equiv \lim_{t\to 0}\gamma(t) = -\frac{1}{\zeta_i^*}\frac{d\Pi_i^*}{dt}(0). \tag{32}$$

The free allocation required to ensure profit-neutrality (to first order) at firm i is equal to the profit lost per unit of emissions.

In a Cournot setting, one can naturally think of the operating profit of firm i as a function of its own output, q_i, all other firms' output, Q_{-i}, and the market price of emissions permits, t. More formally, $\Pi_i(q_i, Q_{-i}, t) = q_i P(q_i + Q_{-i}) - c_i(t)q_i$. The equilibrium profit $\Pi_i^*(t) \equiv \Pi_i(t, q_i^*(t), Q_{-i}^*(t))$ then varies with t according to

$$\begin{aligned}\frac{d\Pi_i^*}{dt} &= \frac{\partial \Pi_i}{\partial t} + \frac{\partial \Pi_i}{\partial q_i}\frac{dq_i^*}{dt} + \frac{\partial \Pi_i}{\partial Q_{-i}}\frac{dQ_{-i}^*}{dt} \\ &= -z_i q_i^* + q_i^* P'(Q^*)\frac{dQ_{-i}^*}{dt},\end{aligned} \tag{33}$$

where $\zeta_i^* = z_i q_i^*$ and the second equality relies on the first-order condition for profit-maximization that $\partial\Pi_i/\partial q_i = 0$. Using this last result in (32), we obtain a simple expression for PNA:

$$\tilde{\gamma}_i = 1 - \frac{P'(Q^*)}{z_i}\frac{dQ_{-i}^*}{dt}. \tag{34}$$

Our next result follows immediately.

Proposition 8 *The first-order PNA for firm i has the following property:*

$$\tilde{\gamma}_i < 1 \Longleftrightarrow \frac{dQ_{-i}^*}{dt} < 0.$$

This result says that, for small t, PNA for firm i is partial if (and only if) the total output of all *other* firms, $Q^*_{-i}(t)$, falls in response to the introduction of emissions trading. Like a monopolist (for which recall $\tilde{\gamma} = 1$), an individual oligopolist faces an increase in marginal cost from the ETS, but, in addition, it also faces a change in its residual demand curve $p = P(q_i + Q^*_{-i}(t))$. Therefore, the PNA for an individual firm $\tilde{\gamma}_i$ is less than unity if its residual demand curve becomes more favorable, $Q^*_{-i}(t) < Q^*_{-i}$, and vice versa.[24] Therefore, it is clear that PNA for each firm *must* be partial if firms in an industry are sufficiently symmetric (in that they all cut output).

Equations (20) and (21) from Proposition 4, together with (34), now give us an explicit formula for PNA at the firm level.

Proposition 9 *The first-order PNA for firm i,*

$$\tilde{\gamma}_i = 2 - \frac{[2 - \sigma_i E(Q^*)]}{[N + 1 - E(Q^*)]} \frac{\sum_{j=1}^{N} z_j}{z_i}. \tag{35}$$

It is clear from this formula that PNA (measured as a proportion of the firm's initial emissions) will typically not be the same across firms. Almost inevitably, a "one-size-fits-all" allocation policy, in which every firm receives the same proportion of freely allocated permits, will lead to overcompensation for some firms and a degree of undercompensation for others. One special case where this proportion *is* constant across firms is when demand is linear and firms have the same emissions intensity (even if they have different costs, and hence market shares). In this case, $\tilde{\gamma}_i = 2/(N+1) \in (0, 2/3]$ for all i, so PNA is positive but partial.

When the firms are symmetric, i.e., have identical costs and emissions intensities, then $\tilde{\gamma}_i = (2 - E(Q^*))/(N + 1 - E(Q^*)) < 1$ for all i, thus confirming our remarks following Proposition 8. More generally, it is not hard to check that $\gamma_i < 1$ if emissions are uniform and firm i is sufficiently 'typical' in the sense that $\sigma_i \in [1/N, 2/(N+1)]$ (for any value of $E(Q^*)$). When firm i's market share is outside that region, it is possible

[24] Note that this holds independently of emissions intensities (although the change in residual demand itself is, of course, a function thereof).

for $\gamma_i > 1$ so firm i's PNA is *larger* than its initial emissions.[25] (A specific example of this is provided in the next subsection, following Proposition 10.)

Differing emissions intensities are another source of differences in PNA across firms. As one would expect, $\tilde{\gamma}_i$ is decreasing in the emissions of other firms and increasing in its own emissions (see (35), (15), and (16)). For example, assuming that demand is linear (so $E(Q^*) = 0$),

$$\tilde{\gamma}_i = 2\left[1 - \frac{1}{(N+1)}\frac{\sum_{j=1}^{N} z_j}{z_i}\right]. \tag{36}$$

Clearly, $\tilde{\gamma}_i > 1$ if $z_i/(\sum_{j=1}^{N} z_j)$ is sufficiently close to 1. At the other extreme, if $z_i/(\sum_{j=1}^{N} z_j)$ is sufficiently close to zero, then $\tilde{\gamma}_i < 0$ since the scheme has a greater impact on firm i's rivals and its strategic position improves to the extent that it actually makes a *higher* profit after the introduction of the ETS.

4.2 Average PNA for an industry

We now examine the level of PNA needed for profit-neutrality for an industry as a whole. This number is more relevant than firm-specific PNA in terms of providing policy guidance on how many permits to freely allocate to firms (and conversely how many to auction), since firms are usually included in an ETS on an industry-by-industry basis.

Recalling the definition of industry-level PNA that $\Pi^*(t) + t\,\gamma(t)\zeta^* = \Pi^*$, and since $\Pi^*(t) = \sum_{i=1}^{N} \Pi_i^*(t)$, we obtain (after Taylor expansion) the first-order PNA

$$\tilde{\gamma} = -\frac{1}{\zeta^*}\sum_{i=1}^{N}\frac{d\Pi_i^*}{dt}(0). \tag{37}$$

Now, since from (32) $d\Pi_i^*/dt = -z_i^* q_i^* \tilde{\gamma}_i$, we can rewrite this as

$$\tilde{\gamma} = \frac{\sum_{i=1}^{N} z_i \sigma_i \tilde{\gamma}_i}{\sum_{i=1}^{N} z_i \sigma_i}. \tag{38}$$

[25] However, it is also clear from (35) and the assumptions (15) and (16), that $\tilde{\gamma}_i$ never exceeds 2.

In other words, PNA for an industry is an average of PNAs for individual firms, weighted by market shares and emissions intensities. Denoting the industry's Herfindahl index by $H = \sum_{i=1}^{N} \sigma_i^2$, equations (38) and (35) yield an explicit formula for PNA for an industry.

PROPOSITION 10 *The first-order PNA for an industry,*

$$\tilde{\gamma} = 2 - \frac{[2 - HE(Q^*)] \sum_{i=1}^{N} z_i}{[N + 1 - E(Q^*)] \sum_{i=1}^{N} z_i \sigma_i}. \tag{39}$$

In principle, $\tilde{\gamma}$ can take on a wide range of values, both positive and negative. For example, it is known (see, for example, Kimmel (1992)) that in a symmetric Cournot oligopoly, a common increase in marginal cost *raises* total profit (in our notation, $\Pi^*(t) > \Pi^*$) if and only if $E(Q^*) > 2$. We can recover this result using (39); at a symmetric equilibrium and assuming uniform emissions intensities, $\tilde{\gamma} < 0$ if and only if $E(Q^*) > 2$. In this case, industry profits increase with the introduction of the ETS, so the industry is (at least weakly) better off even if it has to buy all the permits it needs at the market price. It is also clear from our earlier discussion of firm-level PNA that if firms (and their emissions intensities) are roughly symmetric, then PNA for each firm—and thus for the industry as a whole—is partial.

However, with sufficiently asymmetric firms, it is possible for $\tilde{\gamma}$ to exceed unity. For example, consider a duopoly with uniform emissions intensity that faces a unit-elastic demand curve $P(Q) = K/Q$ (so industry revenue is constant at K), so $N = 2$ and $E = 2$. It is easily checked that $\tilde{\gamma}_1 = 2(\sigma_1 - \sigma_2)$ and hence that $\tilde{\gamma}_1 = -\tilde{\gamma}_2$. With symmetric firms, therefore, PNA is zero for both firms (and for the industry as well), but if $\sigma_1 > \frac{3}{4}$, then $\tilde{\gamma}_1 > 1$ and $\tilde{\gamma}_2 < -1$. The average PNA $\tilde{\gamma} = 2(\sigma_1 - \sigma_2)^2$ exceeds unity if $\sigma_1 > (\sqrt{2} + 1)/2\sqrt{2} \approx 85\%$.

Such examples notwithstanding, the following result shows that reasonable restrictions on the industry guarantee that industry PNA is partial, and indeed low.

PROPOSITION 11 *Provided emissions intensity and marginal cost are positively correlated, we have*

$$\tilde{\gamma} \leq \tilde{\beta} \equiv 2 - \frac{N\,[2 - HE(Q^*)]}{[N + 1 - E(Q^*))]}. \tag{40}$$

Furthermore, (i) if $H \leq 2/(N+1)$*, then*

$$\tilde{\gamma} \leq \tilde{\beta} \leq 2 - NH \tag{41}$$

and (ii) if $H \geq 2/(N+1)$*, then*

$$\tilde{\gamma} \leq \tilde{\beta} \leq \max\{1, E(Q^*)\}\, H. \tag{42}$$

Note that the bounds on $\tilde{\gamma}$ given in Proposition 11 are all independent of emissions intensities (provided they are positively correlated with marginal costs). This result allows us to draw three important conclusions about average PNA.

(1) *PNA is partial for any industry that is sufficiently fragmented in the sense that its Herfindahl index is sufficiently low* (with $H \leq 2/(N+1)$). This conclusion follows immediately from Proposition 11(i) since H is always bounded below by $1/N$, so $\tilde{\gamma} \leq 2 - NH \leq 1$. This result does not depend on the demand curvature $E(Q^*)$.[26]

(2) *PNA is partial for any industry in which firms compete in strategic substitutes (i.e., all firms have downward-sloping best responses).* To see this, recall that firm i's best response is downward-sloping if (17) holds; if (17) holds for all i, we obtain $\sum_{i=1}^{N} \sigma_i(1 - E(Q^*)\sigma_i) > 0$, which may be rewritten as $1 > HE(Q^*)$. Proposition 11(ii) tells us that $\tilde{\gamma} \leq \max\{1, E(Q^*)\}\, H < 1$ if $H \geq 2/(N+1)$. For the case of $H \leq 2/(N+1)$, we know from Proposition 11(i) that PNA is partial (for any value of $E(Q^*)$).[27]

(3) *PNA is bounded above by the Herfindahl index for any industry that has log-concave demand and is sufficiently concentrated* ($H \geq 2/(N+1)$). This is clear since if $E(Q^*) \leq 1$, Proposition 11(ii) says that $\tilde{\gamma} \leq H$. Note that the required degree of concentration ($H \geq 2/(N+1)$) is very low and likely to hold in many cases. This result

[26] At $H = 2/(N+1)$ and with uniform emissions, formula (39) gives $\tilde{\gamma} = 2/(N+1)$, so the bound in (41) is tight at this value of H. With uniform emissions and symmetric market shares (so $H = 1/N$), the formula (39) gives us $\tilde{\gamma} = (2 - E(Q^*))/(N + 1 - E(Q^*))$; this expression approaches 1 as $E(Q^*)$ approaches $-\infty$, so the bound of 1 given by (41) is again tight.

[27] It also follows from this result that in the example we considered after Proposition 10, $\tilde{\gamma} > 1$ only when either firm has an *upward*-sloping best response curve; indeed, one can check directly that this holds for Firm 1.

gives a far tighter bound on PNA since the Herfindahl index is usually below 50%, and *much* below this level for many industries.[28] More generally, provided the industry is not extraordinarily concentrated, the term $E(Q^*)H$—and thus PNA—is likely to be quite low, even when one chooses an estimate of $E(Q^*)$ that is greater than unity. An instance of this is found in the example in the next subsection.

The V-shaped curve in Figure 1 depicts the upper bound on PNA in the case where $E(Q^*) \leq 1$. The Herfindahl index varies between $1/N$ and 1. Between $1/N$ and $2/(N+1)$, PNA is bounded by $B(H) = 2 - NH$ (i.e., the right hand side of (41)); for $H \geq 2/(N+1)$, PNA is bounded by $B(H) = H$. We see from (40) that $\tilde{\beta}$ is a linear function of H. Figure 1 depicts the graph of $\tilde{\beta}$ for two values of $E(Q^*)$. When $E(Q^*)$ is positive, the line has a positive slope ($\tilde{\beta}'$); when $E(Q^*)$ is negative, it has a negative slope ($\tilde{\beta}''$). In all cases, the line passes through the point $(2/(N+1), 2/(N+1))$ and lies below the B curve.

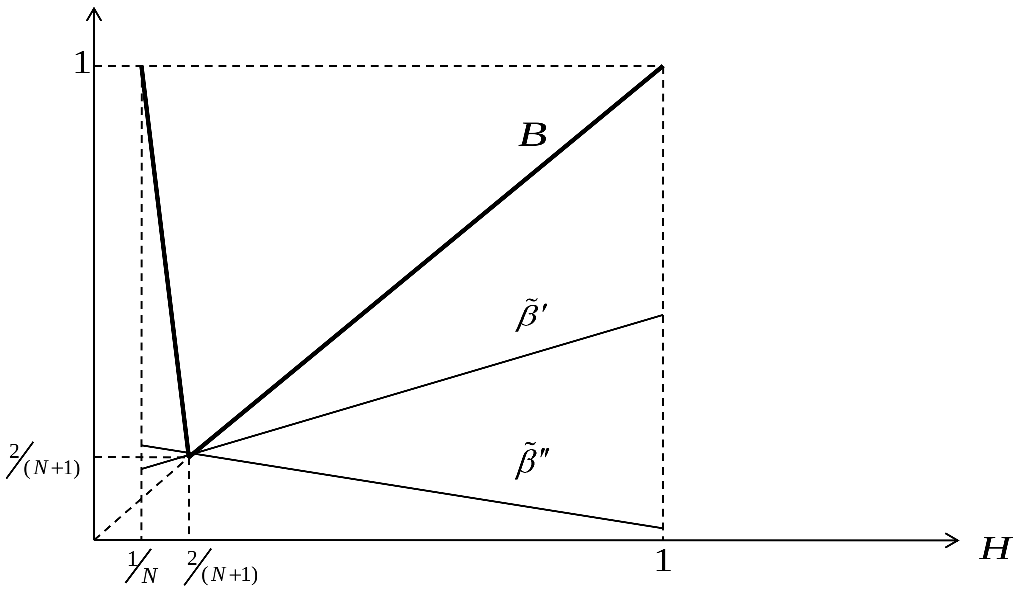

Figure 1: Upper Bound on PNA

[28]The US Department of Justice considers a Herfindahl index between 0.1 and 0.18 as 'moderately concentrated' and anything above 0.18 as 'concentrated'.

These results provide a theoretical benchmark according to which industry-level PNA is partial and low. It is also fairly clear that, whenever an industry's Herfindahl index is relatively low, significant departures from log-concave demand or negative correlation between emissions intensities and market shares will still lead to a low PNA. In the next sub-section, we use an application to the UK cement industry to demonstrate that our basic conclusions on PNA from Proposition 11 are indeed quite robust to different assumptions.

We also show in Appendix C that our main insights, including that industry PNA is partial and low, also apply in a non-local analysis when the permit price is "large".

4.3 Calculating PNA: An example

We now illustrate how our formulae can be used to make indicative calculations of PNA with an application to the UK cement industry, which is part of the EU ETS. In this example, the relevant market definition is at the UK level and it is reasonable to set the number of firms $N = 8$ with a Herfindahl index $H = 0.28$. Note that the concentration condition concentration $H \geq 2/(N+1)$ is therefore satisfied. (Appendix B contains a justification of these values and all other data used in this example.) The two main challenges in using our PNA formula for this application are that demand curvature $E(Q^*)$ is not directly observable and that we do not have detailed information on emissions intensities for each firm in the sector.

As an initial step towards determining PNA, assume that demand is locally log-concave $E(Q^*) \leq 1$ (so the rate of cost pass-through is less than 100%) and also that emissions intensities and marginal costs are indeed non-negatively correlated. Then it follows directly from Proposition 11(ii)) that PNA is bounded above by the industry's Herfindahl index, so $\tilde{\gamma} \leq H = 0.28$. In other words, preserving the industry's profits requires a free allocation of permits covering just 30% of its pre-ETS emissions.

We now consider relaxing both of these assumptions in turn and explore the implications for the calculation of the industry's PNA.

First, a less stringent way of bounding demand curvature $E(Q^*)$ derives from what we call the *elasticity approach.* Observe that we can write

$$E(Q^*) = \left[1 + \frac{1}{\eta(Q)} + \frac{d\log\eta(Q)}{d\log Q}\right]_{Q=Q^*}, \tag{43}$$

where $\eta(Q)$ is the industry price elasticity of demand. With the commonly-made and reasonable assumption that demand elasticity is non-decreasing in price (so $\partial\eta(Q)/\partial Q \leq 0$), we thus obtain an upper bound on demand curvature $E(Q^*) \leq 1 + 1/\eta(Q^*) \equiv \bar{E}$, where $\bar{E} > 1$. If demand has constant elasticity $E(Q^*) = \bar{E}$, but otherwise $\bar{E}$ may be a significant overestimate of the true demand curvature. Calculating $\bar{E}$ is usually straightforward, as it is relatively easy to find estimates of price elasticity $\eta(Q^*)$ for many emissions-intensive industries from previous empirical work. For the UK cement industry, our 'best guess' is $\eta = 0.8$ but we also use a low estimate of 0.5 and a high estimate of 2.0 to check robustness.[29]

Since $\tilde{\beta}$ in Proposition 11 is increasing in $E(Q^*)$,[30] we can write

$$\tilde{\gamma} \leq \tilde{\beta} \leq \bar{\beta} \equiv 2 - N\frac{\left(2 - H\bar{E}\right)}{\left(N + 1 - \bar{E}\right)}. \tag{44}$$

Table 1 displays values for the upper bound $\bar{E}$ on demand curvature, as well as of the upper bound $\bar{\beta}$ on PNA for our range of elasticity estimates. As is consistent with our earlier analysis, PNA (as bounded above by $\bar{\beta}$) is well below unity for all of these estimates, and indeed is always below 50%. We also repeat these calculations for a larger number of firms in the industry (to account for any potential ambiguity over any very small firms not captured in our industry data—since $\bar{\beta}$ also increases with N). The upper-bound estimates of PNA remain well below 100% for these cases, even in the limiting case as we let $N \to \infty$ and so $\bar{\beta} \to \bar{E}H$.

[29] Another way to estimate demand curvature is to use a *cost pass-through approach.* Recall from (24) that the rate of cost pass-through $\kappa = N/\left[N + 1 - E(Q^*)\right]$, so it is possible to back out the value of $E(Q^*)$ from an estimate of cost pass-through. It is easy to check that PNA is partial and low (and sometimes negative) for demand curvatures implied by a very wide range of pass-through rates (both below and above 100%).

[30] Note that $\tilde{\gamma} = \tilde{\beta}$ if emissions is uniform rather than just positively correlated.

Table 1: Upper bounds on PNA in terms of price elasticity (η)

Price elasticity (η)	$\bar{E}$	$\bar{\beta}$ ($N = 8$)	$\bar{\beta}$ ($N = 10$)	$\bar{\beta}$ ($N = 12$)	$\bar{E}H$
0.5 (low estimate)	3.00	0.45	0.55	0.61	0.84
0.8 (best guess)	2.25	0.38	0.43	0.47	0.63
2.0 (high estimate)	1.50	0.31	0.34	0.35	0.42

Second, we relax the assumption that emissions intensities and marginal costs are non-negatively correlated. Recalling the formula for industry-level PNA from Proposition 10, note that its emissions-intensity component can also be written as

$$\frac{\sum_{i=1}^{N} z_i}{\sum_{i=1}^{N} z_i \sigma_i} = \frac{N\bar{z}}{\bar{z} + N\mathrm{cov}(\sigma, z)} \tag{45}$$

where $\bar{z} = \sum_{i=1}^{N} z_i/N$ denotes the average emissions intensity across firms. By definition, the correlation coefficient ρ (of z and σ) is the ratio of $\mathrm{cov}(\sigma, z)$ and the product of the standard deviations of z and σ. We write the standard deviation of z as $\bar{z}\upsilon$, so $\upsilon \geq 0$ is the coefficient of variation of emissions intensities. It is not hard to check that the standard deviation of σ can be written $(\sqrt{HN - 1})/N$. In this way, we obtain

$$\mathrm{cov}(\sigma, z) = \rho\,(\bar{z}\upsilon)\frac{\sqrt{HN - 1}}{N}.$$

Thus we can write the formula for PNA as

$$\tilde{\gamma} = 2 - \frac{[2 - HE(Q^*)]}{[N + 1 - E(Q^*)]}\frac{N}{\left(1 + \rho\upsilon\sqrt{HN - 1}\right)}. \tag{46}$$

This expression allows us to consider departures from the correlation condition $\rho \leq 0$ that we have used to derive theoretical upper bounds on PNA in Proposition 11. (Indeed, note from Proposition 11 that $\tilde{\gamma} \leq \tilde{\beta}$ if $\rho \leq 0$.) In some applications, detailed information on emissions intensities across firms may not be available, but the average emissions intensity across firms $\bar{z}$ may be known and also that emissions intensities are highly unlikely to lie outside a certain range, say $[\bar{z}(1 - s), \bar{z}(1 + s)]$. This information puts an upper bound on the coefficient of variation $\upsilon \leq s$, which in turn implies an upper bound on $\tilde{\gamma}$ in (46).

For the UK cement industry, the information available suggests $s \leq 0.15$ as an upper bound on the coefficient of variation. Table 2 displays estimates of this upper bound on PNA for a range of (maximal) coefficients of variation $\upsilon \leq 0.15$, as well as for the entire range of possible correlation coefficients $\rho \in [-1, 1]$. We assume that demand is log-linear $E(Q^*) = 1$; the PNA estimates would be lower for any strictly log-concave demand curve.

Note first that $\tilde{\gamma} \leq H = 0.28$, as expected whenever the correlation coefficient is negative, and that PNA itself turns negative for very low correlations. Also as expected, the upper bound of the Herfindahl index is tight whenever the coefficient of variation is zero or if the correlation coefficient is zero, as either of these imply that emissions intensities are uniform across firms. Most importantly, note that these upper bounds on PNA always remain low even if the correlation is strongly positive.[31]

This exercise merely confirms something that is fairly clear from (46): PNA remains low, even with positive correlation (i.e., $\rho > 0$), if either there is relatively little variation in emissions intensities (low υ) or if firms' market shares are sufficiently close to symmetric (so H is close to $1/N$).

[31] We also examined the "worst case" scenario for PNA in which the parameter values are all chosen to go as far as possible in the "wrong" direction. In particular, let $E = 3$ for a constant-elasticity demand curve with the low elasticity estimate $\eta = 0.5$, and also let $\rho = 1$ and $\upsilon = 0.15$, so both the correlation coefficient and the coefficient of variation lead to as high a value of PNA as possible. Even in this very extreme case, we find that $\tilde{\gamma} \approx 0.67$, so PNA remains clearly partial and low. Indeed, it is also still lower than the proportion of freely allocated permits in both phases I and II of the EU ETS.

Table 2: Upper bounds on PNA in terms of correlation (ρ) and variation (υ) of emissions intensities

	Correlation ρ				
Variation υ	–1.0	–0.5	0	0.5	1.0
0.00 (uniform intensities)	0.28	0.28	0.28	0.28	0.28
0.05	0.18	0.23	0.28	0.33	0.37
0.10	0.06	0.18	0.28	0.37	0.45
0.15 (maximal variation)	–0.06	0.12	0.28	0.41	0.53

These robustness checks give us confidence that our basic insight that PNA is typically partial and low extends beyond the theoretical benchmarks from Proposition 11. Of course, similar empirical strategies for extending the analysis to log-convex demand curves and other emissions-intensity patterns can be applied to other sectors. For the UK cement industry, a balanced view of this suite of estimates suggests that PNA is indeed likely to be no greater than 25–45%. This implies that the majority of emissions permits could be auctioned whilst preserving industry-level profits in this sector.

5 Conclusion

In this paper we have analyzed, within a canonical theoretical framework, the impact of an emissions trading scheme on output, costs, market shares, profits, and emissions.

We have shown that an ETS leads to more cost-efficient firms gaining market share and a reduction in aggregate industry emissions under the following assumptions: (i) firms' marginal costs are (weakly) positively correlated with their emissions intensities and (ii) the industry faces a log-concave demand function. Moreover, these assumptions guarantee that the profit impact of the ETS (on the industry as a whole), as measured by the profit-neutral permit allocation, is bounded by the Herfindahl index and thus typically low.

We have also provided simple formulae to calculate firm and industry-level PNA that do not rely on assumptions (i) and (ii). These formulae make it plain that the impact of an ETS will differ from one industry to another, depending on market structure, firms' emissions intensities, and demand conditions. They can also be used to provide indicative PNA estimates for an industry. In some cases, firms may require 100% free emissions permits (or more) for profit-neutrality, but our results suggest that such cases are rare. In most cases, a profit-neutral ETS involves a PNA of less (perhaps *much* less) than 50%, so governments can raise substantial revenue from the sale of emissions permits whilst preserving industry profits.

This analysis, we hope, will inform the public discussion of such cap-and-trade schemes as they are implemented in different parts of the world. Our results also provide a natural starting point for further theoretical and empirical studies into ETS design.

Appendix A: Proofs of Propositions 4–7 and 11

Proof of Proposition 4: Let $\hat{q}_i(Q_{-i}, t)$ be the best response of firm i when the other firms are producing Q_{-i} and its marginal cost is $c_i(t)$. Abusing notation, let $MR_i(q_i, Q_{-i})$ denote firm i's marginal revenue when its output is q_i and the other firms are producing Q_{-i}. The first-order condition guarantees that $MR_i(\hat{q}_i, Q_{-i}) = c_i(t)$. Differentiating this equation by t and evaluating it at $t = 0$, we obtain

$$\frac{\partial \hat{q}_i}{\partial t} = \frac{z_i}{\partial MR_i / \partial q_i} = \frac{z_i}{2P' + q_i P''} \tag{47}$$

Differentiating the same equation by Q_{-i}, we obtain

$$\frac{\partial \hat{q}_i}{\partial Q_{-i}} = -\frac{\partial MR_i / \partial Q_{-i}}{\partial MR_i / \partial q_i} = -1 + \frac{P'}{2P' + q_i P''}. \tag{48}$$

At equilibrium, $\hat{q}_i(Q^*_{-i}, t) + Q^*_{-i} \equiv Q^*$. Differentiating this with respect to t and using (47) and (48), we obtain

$$\frac{dQ^*_{-i}}{dt} = \frac{dQ^*}{dt}\left[2 - \sigma_i E(Q^*)\right] - \frac{z_i}{P'}. \tag{49}$$

Summing this equation across firms gives us

$$(N-1)\frac{dQ^*}{dt} = \frac{dQ^*}{dt}[2N - E(Q^*)] - \frac{\sum_{i=1}^{N} z_i}{P'}. \tag{50}$$

Rearranging now yields (20). Using (20) to substitute for P' in (49), we obtain (21). Since $dq_i^*/dt = dQ^*/dt - dQ_{-i}^*/dt$,(22) may be derived from (20) and (21). QED

Proof of Proposition 5: Using (26) and (29), we obtain

$$\frac{dH}{dt} = \frac{1}{NQ^*}\frac{dQ^*}{dt}\left[-NH - N(1 - HE(Q^*)) + (N + 1 - E(Q^*))\frac{\sum_{i=1}^N \sigma_i z_i}{\sum_{i=1}^N z_i/N}\right].$$

Since $dQ^*/dt < 0$, $dH/dt \geq 0$ if the term in the square brackets is non-positive. Using the fact that $\sum_{i=1}^N \sigma_i z_i \leq \sum_{i=1}^N z_i/N$ (see (19)), the term in the square brackets is bounded above by

$$(HN - 1)(E(Q^*) - 1).$$

This is non-positive since $E(Q^*) \leq 1$ and $H \geq 1/N$. QED

Proof of Proposition 6: Using (26) and (12), we obtain

$$\frac{d\underline{c}^*}{dt}(0) = \frac{1}{NQ^*}\frac{dQ^*}{dt}\left[N(E(Q^*) - 1)\sum_{i=1}^N \sigma_i c_i - N\sum_{i=1}^N c_i + (N + 1 - E(Q^*))\frac{\sum_{i=1}^N c_i z_i}{\sum_{i=1}^N z_i/N}\right].$$

Since $dQ^*/dt < 0$, $d\underline{c}^*/dt \leq 0$ if the term in the square brackets is non-negative. Using the fact that $\sum_{i=1}^N c_i z_i/(\sum_{i=1}^N z_i/N) \geq \sum_{i=1}^N c_i$ (see (18)), the term in the square brackets is bounded below by

$$(1 - E(Q^*))\left(\sum_{i=1}^N c_i - N\sum_{i=1}^N \sigma_i c_i\right).$$

This is nonnegative because both terms in the product are nonnegative. Note that the second term in this product is nonnegative because σ_i and c_i are respectively decreasing and increasing in i. QED

Proof of Proposition 7: Since $dQ^*/dt < 0$ (see (20)), $d\zeta^*/dt \leq 0$ if $dz^*/dt \leq 0$. For case (a), we have already established that $dz^*/dt \leq 0$ in the main part of the paper. So we turn to case (b). It suffices to show that $\sum_{i=1}^N z_i[d\sigma_i/dt] \leq 0$. By (26),

$$\sum_{i=1}^N z_i\frac{d\sigma_i}{dt} = \frac{1}{NQ^*}\frac{dQ^*}{dt}\left[N(E(Q^*) - 1)\sum_{i=1}^N \sigma_i z_i - N\sum_{i=1}^N z_i + (N + 1 - E(Q^*))\frac{\sum_{i=1}^N z_i^2}{\sum_{i=1}^N z_i/N}\right].$$

We require the term in the square brackets to be nonnegative. Using the fact that $\sum_{i=1}^{N} z_i^2 \geq (\sum_{i=1}^{N} z_i)^2/N$, the term in the square brackets is bounded below by

$$(1 - E(Q^*)) \left(\sum_{i=1}^{N} z_i - N \sum_{i=1}^{N} z_i \sigma_i \right).$$

This is nonnegative since both terms in the product are nonnegative (the second by (19)).

QED

Proof of Proposition 11: By (15), we have $2 - HE(Q^*) = \sum_{i=1}^{N} \sigma_i (2 - \sigma_i E(Q^*)) > 0$. Using (19), we obtain (40) from (39). We may rewrite $\tilde{\beta}$ as

$$2 - NH - N(N+1) \frac{[2/(N+1) - H]}{[N + 1 - E(Q^*)]}. \tag{51}$$

If $H \leq 2/(N+1)$, this term is always less than $2 - NH$, so we obtain (i). (Note that $N + 1 - E(Q^*) > 0$ by (16).) If $H \geq 2/(N+1)$, this term increases with $E(Q^*)$, so we may replace $E(Q^*)$ with $\widehat{E} = \max\{1, E(Q^*)\}$ to obtain

$$\tilde{\gamma} \leq \tilde{\beta} \leq \hat{\beta} \equiv 2 - \frac{N \left[2 - H\widehat{E} \right]}{\left[N + 1 - \widehat{E} \right]}. \tag{52}$$

Since $E(Q^*)$ obeys (16), we also have $N + 1 - \widehat{E} > 0$ and since it obeys (15) we obtain $2 - H\widehat{E} = \sum_{i=1}^{N} \sigma_i (2 - \sigma_i \widehat{E}) > 0$. Given these and the fact that $\widehat{E} \geq 1$ by construction, it is easy to check that $\hat{\beta}$ (as defined in (52)) is increasing in N and has a supremum of $H\widehat{E}$.

QED

Appendix B: The UK cement industry

There are five certified types of cement: Portland cement, Portland blast furnace cement, sulphate-resisting cement, masonry cement, and Portland pulverized fuel ash cement—which we group together because they are manufactured with essentially the same process (Environment Agency, 2005). The UK cement market is dominated by the four members of the British Cement Association: Lafarge Cement UK (previously Blue Circle), Castle Cement (owned by Heidelberg Cement), Cemex (previously Rugby Cement) and Buxton Lime Industries. These four firms collectively produce around 90% of the cement sold in the UK, with approximate market shares of 40%, 25%, 20% and 5% (Environment Agency, 2005). Imports from four other firms (all manufacturing within the EU and subject to the EU ETS) supply the remainder. This gives a Herfindahl index of around $H = 0.28$ with a number of firms $N = 8$.

Estimates of the price elasticity of demand for cement in the UK do not seem to be readily available. Jans and Rosenbaum (1997) find an average elasticity of demand of 0.80 for cement industry in the U.S. More recently, Ryan (2005) finds an elasticity of 2.95 from US market-level data on prices and quantities. While noting this is a rather high estimate, he argues that it is consistent with data on profit margins and plant costs. Finally, Röller and Steen (2005) find a short-run elasticity of 0.46 and a corresponding long-run elasticity of 1.47 for the Norwegian market. For our calculations, we employ price elasticities of 0.5 (low), 0.8 (best guess) and 2.0 (high).

The common standard of measurement for carbon emissions intensities in cement production is kilograms of CO_2 per ton of Portland cement equivalent (tPCE). Emissions intensities are driven by a combination of factors, including plant size, plant age, processing technology and fuel mix. For 2005, the British Cement Association stated in its 2006 Performance Report that its four members had an average emissions intensity of 822 kgCO_2/tPCE, but the report does not contain any data for individual firms. We have obtained emissions intensities for the two largest firms from other sources, but have not been able to obtain figures for the other firms in the industry. Lafarge Cement UK, the market leader, reports an emissions intensity of around 770 kgCO_2/tPCE for 2005, while Castle Cement's emissions intensity was around 820 kgCO_2/tPCE (see Lafarge Cement UK – 2005 Environmental Statement, and Castle Cement – 2007 Sustainability Report respectively).

We found no indication from cement industry sources of large differences in emissions intensities across firms. It seems likely that firms in the sector have emissions intensities within the range 700-940 $kgCO_2/tPCE$, implying that the larger part of the marginal cost impact of the EU ETS is commonly experienced. Amongst other things, this implies that the coefficient of variation of emissions intensities (that is, their standard deviation divided by the average emissions intensity in the industry of around 820 $kgCO_2/tPCE$) is likely to be no greater than around 0.15. Our evidence, though limited, is also consistent with the assumption that emissions intensities and market shares are (weakly) negatively correlated. In any case, it seems unlikely that this correlation coefficient takes an extreme value on either the positive or negative side in the UK cement industry.

Appendix C: The impact of the ETS for large t

Our analysis in the main text of the impact of an ETS focuses on its first-order impact (for small t). We show here that our main results are preserved when the permit price is not small. In particular, when demand is log-concave, PNA is bounded by the Herfindahl index and thus likely to be lower (perhaps *much* lower) than 50%.

We assume that the permit price is $T > 0$, and for all t in $[0, T]$, the Cournot equilibrium exists and varies smoothly with t. It follows from (4) that the industry PNA may be written as

$$\gamma(T) = \frac{1}{T\zeta^*(0)} \left[\Pi^*(0) - \Pi^*(T) \right]. \tag{53}$$

Since $\Pi^*(T) = \Pi^*(0) + \int_{t=0}^{T} [d\Pi^*(t)/dt] \; dt$ and defining $\lambda(t) = - [d\Pi^*(t)/dt] / \zeta^*(t)$, we obtain from (53)

$$\gamma(T) = \frac{1}{T\zeta^*(0)} \int_{t=0}^{T} \lambda(t)\zeta^*(t)dt \tag{54}$$

Using essentially the same arguments that led to the formula for $\tilde{\gamma}$ (see Proposition 10), we find that

$$\lambda(t) = 2 - \frac{[2 - H(t)E(Q^*(t))]}{[N + 1 - E(Q^*(t))]} \frac{\sum_{i=1}^{N} z_i(t)}{\sum_{i=1}^{N} z_i(t)\sigma_i(t)}. \tag{55}$$

It follows that for any positive scalar K, $\gamma(T) < K$ provided (i) $\lambda(t) < K$ and (ii) $\zeta^*(t)$ is decreasing in t for all t in $[0, T]$. It is possible to bound $\lambda(t)$—and thus $\gamma(T)$—by mimicking the arguments of Section 4.3 (and, in particular, Proposition 11). Instead of covering all the possible scenarios, we here simply give a flavor of how the argument works by focusing on two important cases that guarantee partial PNA.

Conditions (i) and (ii) hold in the following cases:

Case A. The demand function obeys $[N+1-E(Q)] > 0$ for any $Q > 0$. All firms operate the same technology, so (in symmetric equilibrium) each firm produces at the same output level using an identical input (including emissions) mix.

Case B. The demand function is globally log-concave, i.e., $E(Q) < 1$ for $Q > 0$. Market concentration before the introduction of the ETS obeys $H(0) \geq 2/(N+1)$. For any t in $[0, T]$, the marginal costs $c_i(t)$ are positively correlated with the emissions intensities $z_i(t)$; i.e., (18) holds at any $t \in [0, T]$.

Solving Case A is straightforward. Equilibrium output $Q^*(t)$ falls with t since $[N+1-E(Q)] > 0$ (which can be proved using essentially the same arguments that led to (20)). Standard revealed preference arguments guarantee that the optimal emissions intensity at firm i, $z_i(t)$, decreases with t. Since firms are identical, $z_i(t)$ also equals the average emissions intensity, $z^*(t)$. It follows that total emissions $\zeta^*(t) = z^*(t)Q^*(t)$ are decreasing in t, so condition (ii) is satisfied. Furthermore, with a symmetric equilibrium, $(\sum_{i=1}^N z_i(t))/(\sum_{i=1}^N z_i(t)\sigma_i(t)) \equiv 1$ and $H(t) \equiv 1/N$, so

$$\lambda(t) = \frac{2 - E(Q^*(t))}{[N+1-E(Q^*(t))]} \leq \frac{2 - E_m}{[N+1-E_m]} \tag{56}$$

where $E_m = \min_{0 \leq t \leq T} E(Q^*(t))$. Note that the bound $M \equiv (2-E_m)/(N+1-E_m)$ is strictly less than unity. Therefore, it follows from (54) that $\gamma(T) \leq M$. We conclude that, *under the assumptions of Case A, the introduction of the ETS reduces output, emissions intensity, and emissions; PNA is partial and bounded by M.*

A similar conclusion holds for Case B, though the proof is less straightforward. In this case, *the introduction of the ETS lowers industry output, emissions intensity, and total emissions; formally, $Q^*(T) < Q^*(0)$, $z^*(T) \leq z^*(0)$, $\zeta^*(T) < \zeta^*(0)$. Industry PNA, $\gamma(T)$, obeys $\gamma(T) \leq H(T)$, where $H(T)$ is the Herfindahl index at the post-ETS equilibrium. In particular, industry PNA is partial.*

To establish this claim, note firstly that since $E(Q) < 1$, the inequality $N+1-E(Q) > 0$ holds and guarantees that equilibrium output $Q^*(t)$ falls with t. Revealed preference arguments guarantee that $z_i(t)$ decreases with t. Thus, *average* emissions intensity $z^*(t)$ falls with t if firms with (weakly) lower emissions intensity gain market share with increasing t. This is true because, with the correlation assumption, firms with lower emissions will typically have lower marginal costs and these firms gain market share when t increases and demand is log-concave.

In formal terms, we need to check that the proof of Proposition 7 does not hinge on $t = 0$ (and indeed it does not); its conclusion holds so long as demand is log-concave and market shares are negatively correlated with emissions intensities at any value of t in $[0, T]$. So we know that $\zeta^*(t)$ decreases with t. Similarly, by replicating the proof of Proposition 5, we may conclude that $H(t)$ decreases with t in $[0, T]$. The negative correlation between market shares and emissions intensities follows from the assumption that marginal costs and emissions intensities are positively correlated (see discussion preceding (19)).

Finally, to obtain the bound on industry PNA, use the result that the Herfindahl index $H(t)$ increases with t, and note that essentially the same argument that guaranteed (42) in Proposition 11(ii) can be used to show that $\lambda(t) \leq H(t)$. (This argument requires $H(t) \geq 2/(N+1)$, which is true since $H(t)$ increases with t and we assume that $H(0) \geq 2/(N+1)$.) Therefore, $\lambda(t) \leq H(T)$ for all t in $[0, T]$. Using (54), we obtain $\gamma(T) \leq H(T)$.

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