Investment in Electricity Generation Under Emissions Price Uncertainty: The Plant-Type Decision

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ABSTRACT:

This paper investigates the effects of uncertain emissions prices on the plant-type investment decision of an electricity generating firm. It is assumed that the firm faces an investment deadline and is deciding between "clean" and "dirty" generating alternatives, with the dirty option being more profitable in the absence of emissions regulation. Two models are presented in which the policy variable, the carbon dioxide (CO_2) price, is assumed to evolve according to two different stochastic processes, each representing a different degree of possible future government intervention. In the first, the emissions cap is assumed fixed and certain, and CO_2 permit prices evolve according to a geometric Brownian motion, and in the second it is assumed that the government may alter the cap at some point in the future, and CO_2 prices are modelled as following a combined Poisson-geometric Brownian motion. The models employ real options analysis and the firm's decision is couched as an optimal stopping problem in which the continuation region is characterized by the default decision of replacing the dying coal plant with a new dirty unit. The models are solved for the critical permit price, τ^*, at which the construction of a clean plant rather than a dirty one becomes optimal, and the dependence of the models' solutions on the parameter values chosen is demonstrated. The results indicate that uncertainty leads to a delay in investment generally, and a reduction in clean-unit construction. It is shown that this delay is more pronounced in the jump-diffusion scenario, when emissions caps can be altered in the future, than in the geometric Brownian motion setting, when the emissions cap is assumed fixed.

Contents

1 Introduction

This paper focuses on the effects of uncertain carbon dioxide (CO_2) prices on the plant-type investment decision in the electricity generating industry. The data is from the US, and there is a slight US bias in the writing, but the models and their implications are applicable more widely. The motivation for this research relates to the multiplicity of government objectives surrounding regulation in the electricity generation (see, e.g. Helm (2002) and Helm et al. (2003)) and the uncertainty it creates for firms in the industry. Energy policy makers are faced with the task of balancing the opposing objectives of economic growth, security of supply, and environmental protection. The objective of economic growth has policy makers seeking low electricity prices, while security of supply objectives imply higher electricity prices to incentivize capacity investment and, in the US, support for coal-burning technologies, given the nation's abundant coal resources. These concerns must be balanced with those related to the environmental impacts of electricity generation through fossil fuel combustion, the regulation of which is likely to cause higher output prices, reduced capacity investment, a move away from coal combustion (unless carbon capture and sequestration (CCS) technologies become viable relatively soon), or some combination of all of these things. The result is uncertainty from the firm's perspective regarding future emissions regulation.[1] This paper addresses the effects of this uncertainty, modelled here using uncertain future emissions prices, on electricity generating firms' capacity investment decisions and shows the usefulness of real options analysis in modelling these effects.

In what follows, firms are assumed to operate under a cap-and-trade CO_2 allowance scheme. No such scheme currently exists in the US, though CO_2 permits have been traded in the European Union (EU) under the EU Emissions Trading Scheme (EU ETS) since 2005, and sulphur dioxide (SO_2) allowances have been traded in the US since 1995.[2] The emissions price provides a useful means for introducing environmental policy uncertainty to an investment model, and yields insight regarding the effects of this uncertainty. It could also be argued that US electricity generating firms expect some form of CO_2 regulation is impending, and given the success of the SO_2 programme and the political popularity of the EU ETS, a cap-and-trade scheme does not appear unlikely.

The problem facing electricity generating firms is as follows. They know that given current market conditions, electricity generation via fossil fuel combustion, and particularly with coal, is significantly cheaper than generation with renewable sources.[3] They are aware that the nation's livelihood effectively rests on their successful provision of electricity, that there exists a political fear of resource dependency and that coal is available in vast quantities in the US, but they are also aware of the increasing pressure for the approval of a unified environmental programme regulating the emissions of greenhouse gases, and specifically carbon dioxide, and as a result, they are faced with uncertain future emissions prices. As demand rises and existing plants age, firms considering new investment must decide whether to continue building "dirty" combustion units, allowing generation at a lower thermal cost per

[1] We note that policy delay and lack of commitment may, indeed, be an optimal response for the regulator.
[2] See Stavins (2003), Joskow et al. (1998), and Tuthill (2007, 2008) for further discussion of the US SO_2 scheme.
[3] In 2006, coal's average price per mmBtu was just 169 cents, or 24 percent of that for natural gas, whose average price was 694 cents per mmBtu, and 27 percent of petroleum's price of 623 cents per mmBtu (EIA, 2007a).

kilowatt hour, or rather to build new "clean" generating units, which incur a higher thermal cost per kilowatt hour, but save the costs associated with future CO_2 emissions. Thus, the problem addressed in this paper is that of an electricity generating firm's plant-type investment decision under uncertainty over future emissions prices. I show that emissions price uncertainty leads to a reduction in clean generating capacity construction and a delay in capacity investment generally, and I provide numerical analyses for specific plant-type investment choices. The results indicate that the combined objectives of emissions reduction and the maintenance of a secure electricity supply in the US may be best achieved through the establishment of a credible and certain CO_2 scheme.[4]

The irreversible nature of power plant investment leads to an opportunity cost associated with the uncertain future cost of generation as determined by uncertain future emissions prices, rendering the classical net present value (NPV) theory of investment invalid. Instead, the opportunity to invest in a clean electric power plant can be viewed as a financial call option, endowing the firm with the right, but not the obligation, to invest a certain amount today in return for a power plant of uncertain value. The two models of the plant-type investment decision presented in this paper are therefore based on real options analysis (see Dixit and Pindyck (1994)). The source of uncertainty in each is the CO_2 allowance price and each uses a different stochastic process for its evolution to reflect different policy assumptions. The first assumes that the regulator establishes an emissions cap and relinquishes the right to adjust it, and that permits are then traded freely between firms with prices that effectively evolve randomly according to a geometric Brownian motion (GBM). The second assumes the regulator retains the right to adjust the emissions cap and permit prices are modelled as following a mixed Poisson-geometric Brownian motion (or jump-diffusion) process. Both models view the firm's decision as an optimal stopping problem and both are solved via dynamic programming.

The remainder of this paper is structured as follows. The theoretical background for the investment models and a review of the related literature are provided in Section 2. I then present the models in Section 3, and implement and analyse them in Section 4. Section 5 assesses the dependence of the results in Section 4 on the parameter assumptions, and Section 6 illuminates the conclusions.

2 Theoretical Background and Literature Review

2.1 Theoretical Background

Orthodox neoclassical investment theory relies on the calculation of the net present value of a marginal unit of capital and posits that capital investment should continue until the cost of the marginal unit of capital is just equal to its net present value. This theory, while not ideal for the modelling of the generating-unit investment decision for the reasons described below, provides the background for investment analysis via real options models. Dixit and Pindyck (1994) find two equivalent approaches in the investment literature that deal with the issues of calculating the value and the cost of a marginal unit of capital. One is due to Jorgenson (1963) and the other to Tobin (1969).

[4] We do not claim that these policies or objectives are optimal from the perspective of the regulator, as we do not perform a welfare analysis of their impact.

Jorgenson (1963) postulated that the demand for capital is determined by maximizing net worth and results from equating the marginal product of capital with its "user cost". This user cost is a function of the purchase price of capital, interest rates, depreciation rates, and relevant taxes, and is used to calculate K_t*, the optimal level of capital stock to which the actual capital stock is assumed to converge through an investment lag process.

Tobin's (1969) formulation, on the other hand, compares the market value of a marginal investment to its purchase cost, where the market value of the investment is estimated as the discounted value of the future profits the investment would provide.[5] Tobin's q is defined as the ratio of the value of the marginal investment to its purchase price (or replacement cost), which produces the investment rule "invest if q>1, don't invest if q<1."[6]

Econometric models based on these net-present-value approaches have not been very successful at explaining or predicting investment behaviour (Pindyck (1991b)). The more recent real options literature incorporates uncertainty into the investment decision and notes one of the main reasons for the lack of empirical success of the neoclassical models is the neglect of two basic characteristics of most investment expenditures: irreversibility and the flexibility of investment timing.[7] When investments are irreversible and the investment decision can be delayed, the neoclassical net present value investment rules are not necessarily correct.[8]

An investment is considered irreversible when the investment expenditure can be interpreted as a sunk cost, i.e. when the investment is firm- or industry-specific, as is the case with an electric power plant. Irreversible capital can not be easily converted for another production process, and the firm can not easily recover its expenditure should future market conditions deteriorate. It is irreversibility that makes an investment decision especially sensitive to risk and uncertainty over future values and cash flows, and it is therefore the interaction of irreversibility and uncertainty (or, more precisely, future downside risk) that forms the cornerstone of this more recent investment under uncertainty literature.

This has led to the incorporation of the notion of option value into the investment literature, with the option to invest in a new project being interpreted as analogous to a perpetual call option, giving the firm the right, but not the obligation, to pay a certain amount (the investment expenditure) in order to receive an asset (in this case a power plant) of uncertain future value. Thus defined, the value of the investment option is gleaned from the existence of future downside risk and the opportunity cost of investing now rather than waiting for more information. From this notion of option value, two separate but related strands of the investment literature have emerged. The first is associated with the environmental economics literature and stems from work by Henry (1974) and Arrow and Fisher (1974), emphasizing the idea that irreversibility can bring a positive value to preservation (e.g. of a natural resource), or, equivalently, of postponing developing (investment). The second began with the static models of option value by Cicchetti and Freeman (1971) and has evolved into the

[5] If the ownership of the investment is traded on a secondary market, the investment's value is determined there.
[6] See Romer (1990) for a more detailed discussion.
[7] Note that the empirical success of real options models has also been mixed, though Ishii and Yan (2004, see below) found uncertainty regarding deregulation to have played a "substantial" role in US electricity generating investment, and Bulan (2005) notes that real options investment behaviour has been found more prevalent at the project-level (as opposed to aggregate industry or firm-level).
[8] See, for example, Hanemann (1989), Pindyck (1991), and Dixit and Pindyck (1994).

perhaps more well-known dynamic option value literature surrounding Pindyck (1991), and Dixit and Pindyck (1994).
The environmental economics literature surrounding investment under uncertainty is not the focus of this paper, but it supports the assumption that electricity generating firms are uncertain over future environmental regulations. This literature concerns itself more with the decisions of the government, or other regulating body, than with those of the firm, and looks mainly at the question of when and whether to develop (or pollute) given that both development (pollution) and environmentally friendly capital investments are irreversible and that the value of the future benefits of preservation (not emitting) and costs of development (polluting) are uncertain. It is also assumed that the agents learn about these future costs and benefits as time passes. The option value in this literature is tied to the concept of learning. It can be interpreted as the conditional value (conditional on not developing initially) of perfect information about the future costs and benefits of preservation and is related to the value of flexibility (i.e. the value of retaining choices to make at a future date) and not just to irreversibility.[9] In this literature, the objective is to balance these uncertain future costs and benefits in order to create an environmental policy while learning over time.

In two early examples, Henry (1974) and Arrow and Fisher (1974) modelled the decision of whether to develop a patch of land and showed that if the social benefit function is linear, increasing the uncertainty over future costs and benefits leads to a higher value of preservation in the present. Kolstad (1996) and Pindyck (2000), however, note that these early studies did not account for the irreversibility of the capital investments required to meet some environmental standards. They model the decision of when and whether to institute an environmental emissions policy, noting the interaction of the irreversibility of environmental damage with the irreversibility of the capital investments required to reduce emissions and find that greater uncertainty about the future costs and benefits of emissions regulation lead to a higher threshold for policy adoption. Thus, the environmental economics strand of the investment under uncertainty literature indicates that the when facing uncertain environmental costs, emissions policy adoption may be slow and tentative, leaving firms uncertain over future environmental regulation.

From the electricity generating firm's perspective, future emissions prices, particularly as they are dictated by government-established caps, therefore remain uncertain and the option valuation strand of the investment under uncertainty literature becomes more relevant to the plant-type investment decision. This literature is summarized well in Pindyck (1991) and Dixit and Pindyck (1994), and is based on the assumption that capital investments are irreversible and can be delayed, leading to the above-mentioned analogy of a firm's option to invest in a plant or project as a perpetual call option. In most of the literature, a firm is presented with the opportunity to invest in a project that will yield an uncertain cash flow in the future, and must decide if and when to invest. The future value of the project is unknown because of uncertainty over one or more of the underlying variables, generally modelled as evolving according to some variation of a random walk.

For an electricity generating firm, the option to invest has a value itself because of the uncertainty over the plant's future value, which, here, is determined by the CO_2 price. The firm choosing to invest today faces two possibilities: market conditions tomorrow could remain favourable for the investment, or they could deteriorate. Given the irreversibility assumption, however, a firm can not disinvest if the market moves adversely. Thus, a firm

[9] See Hanemann (1989), Kolstad (1996), Henry (1974), and Arrow and Fisher (1974) for more on this topic.

investing in a dirty plant today may learn tomorrow that CO_2 regulation will become tighter and allowances will command a higher price per ton, and a clean plant may have been more profitable in retrospect. Alternatively, they may invest in a clean plant today, expecting stringent CO_2 regulation and sufficiently high allowance prices, only to learn that regulation will be delayed or that CO_2 prices will be relatively low. It is the existence of this downside risk and the opportunity cost it creates of investing today rather than waiting for further information that establishes the value of the investment option. When the firm invests in a new plant, it exercises its option, giving up the opportunity to wait for the revelation of more information about future emissions prices. The crux of the option value literature is that this lost option value should be interpreted as an opportunity cost of investment, and ignoring it can lead to investment rules that are far from optimal.[10]

The real options method is best explained in the context of an example, along the lines of those used by Pindyck (1991) and Dixit and Pindyck (1994). Assume a firm is deciding when to invest in a project of uncertain future value V, and V evolves according to the following geometric Brownian motion

$$dV = \alpha V dt + \sigma V dz ,$$

where α is the expected growth rate of the value of the project, σ measures the volatility of the value of the project, and dz is an increment of the Wiener process.[11] Thus, the current value of the project is known, and information about future values arrives continuously, but the actual future value is always uncertain.

The models presented in Section 3 are based on the assumption that the firm is trying to maximize the value of the investment opportunity over time, and they employ the dynamic programming approach[12,13] to solve for the optimal investment rule. If $F(V)$ represents the value of the option to invest as it depends on V, the value of the project, then the firm solves

$$F(V) = \max E_t\left[(V_t - I)e^{-\mu T}\right],$$

where I is the investment cost, μ is the discount rate, and T is the future time that the investment is made. Because the investment opportunity yields no actual cash flow while it is being held, its value is derived strictly from its expected capital gain. Thus, the Bellman equation for the problem is

$$\mu F = \frac{1}{dt} E_t dF ,$$

which, through the application of Ito's Lemma (Ito (1951)), leaves us with a differential equation for the value of the option to invest. This differential equation can then be solved subject to the relevant boundary conditions to yield the critical V^* that induces investment and the value of the option to invest.

[10] See, for example, Brennan and Schwartz (1985), McDonald and Siegel (1986), Pindyck (1991), Dixit and Pindyck (1994).

[11] The Wiener process is discussed further in the following section.

[12] See Dixit (1990) for an exposition of the methods of dynamic programming.

[13] Dixit and Pindyck (1994) note that when the risk characteristics of the uncertain value can be replicated by a dynamic spanning portfolio of other assets, contingent claims analysis (an option pricing method) can also be used as a solution method. When the firm is risk-neutral, the contingent claims and dynamic programming methods yield identical results (Pindyck (1991)).

The models in Section 3 address an electricity generating firm's plant-type investment decision as an optimal stopping problem. Optimal stopping problems are a subclass of dynamic programming problems that involve a choice to either "stop" or "continue" in each period. In this case, the construction of a dirty plant remains optimal in the continuation region, while constructing a clean plant becomes optimal in the stopping region. The boundary between the regions is created by the curve representing the critical values of the state variable (in this case τ^*, the critical CO_2 permit price) at which the firm's optimal choice switches from "continue" (build dirty) to "stop" (build clean). Because the boundary is unknown and must be found as part of the solution, these problems are also known as free-boundary problems.

The formulation of the models presented below is somewhat different from those that are standard in the literature in that the continuation region of the optimal stopping problem here is where the dirty investment (as opposed to no investment) is optimal. Thus, in the models below, the firm has already decided it would like either to replace retiring capacity or expand existing capacity by a known number of MWs, and is not, therefore, deciding whether or not to invest in a given unit, but rather whether the new unit they would like to construct should be clean or whether they should build the default (and currently more profitable) dirty unit. Before presenting the models, a discussion of some of the relevant related literature is provided.

2.2 Models and Applications in the Literature

Much of the applied literature involving investment under uncertainty investigates the effects of output demand and product price uncertainty or investment cost uncertainty on the decision of whether or not to invest in a specific project.[14] In this paper, however, the focus is more specifically on production cost uncertainty associated with future emissions regulations, and I am interested in the impact this uncertainty has on the choice between alternative generating technologies. Little work has been done to address the issue of emissions price uncertainty on investment, especially within the electricity generating industry, though some work has modelled the effects of other types of policy uncertainty. For example, Hassett and Metcalf (1999) investigated the effects of uncertain tax policies on both firm-level and aggregate investment, and Ishii and Yan (2004) modelled the effects of uncertainty over future regulatory restructuring on capacity investment in US electricity generation. Herbelot (1994) and Insley (2003) focused on environmental policy and uncertainty and took advantage of the natural experiment provided by the allowance scheme instated in the US by Title IV of the Clean Air Act Amendments of 1990 to examine the abatement investment decisions of affected firms. Most related to the present work is a paper by Blyth et al. (2007), discussing an International Energy Agency (IEA) model that addresses capacity investment under uncertain emissions policy.

Hassett and Metcalf (1999) concerned themselves with the question of whether or not random tax policy discourages investment. They did so by analyzing the effects of a mean-preserving spread in investment tax credits (ITCs) for new capital, and they focused on the timing and quantity of investment, without addressing the question of investment type. The most relevant contribution of their paper to this research is related to their modelling of uncertainty. Hassett and Metcalf (1999) modelled their ITC uncertainty with two alternative

[14] See, for example, McDonald and Siegel (1986), Pindyck (1993), Pindyck (1991).

processes: a geometric Brownian motion (GBM) and a combined GBM and Poisson jump process. They argued that while output and asset prices may follow a process that is well-described by GBM, tax policies, and specifically tax credits, are better characterized by discrete jumps. Their findings show that the impact of tax policy uncertainty on investment depends heavily on whether the stochastic process used to model uncertainty is mean stationary or not. They found that when modelling ITCs via GBM, which is not a mean stationary process, increased uncertainty slows investment (i.e. requires a higher critical output price to induce investment), while when using the mean-stationary combined GBM and Poisson jump process (which, they argue, is more accurate for modelling ITCs), increasing uncertainty has the opposite effect. In this paper, I consider the impact of emissions policy uncertainty on electricity generating firms' investment decision with the policy variable modelled as following both the more standard GBM and a modified combined GBM and Poisson process, allowing for either positive or negative jumps.

Ishii and Yan (2004) focused on the effects of uncertainty related to future deregulation activity in the US electricity generating industry. They argue that, by creating an option value and an incentive for firms to wait for further information about the future operating environment, regulatory uncertainty can contribute to an "investment slowdown".[15] They analyse this claim empirically with data covering the period 1996-2000 using a Tobit regression model on aggregate state-level investment and a more structural model explicitly incorporating the option value effect using firm-level generation investment data. They found that states that eventually enacted restructuring legislation saw lower investment levels in the years prior, and that the option value created by uncertain regulatory policy was largest 2-3 years prior to actual restructuring activity. Their structural models show that the NPV investment rule did not adequately describe investment activity, while their model based on real options fit the data best. They conclude that regulatory policy uncertainty played a "substantial" role in US electricity generation since 1996.

As noted, Herbelot (1994) and Insley (2003) both investigated the abatement investment decisions of US electricity generating firms affected by the CAAA 1990 and its tradable allowance scheme. Herbelot (1992) argued that under the SO_2 scheme, firms had the options to 1) retrofit high-sulphur-coal-fired units in order to accommodate the combustion of low-sulphur coal, 2) install scrubbers in their smokestacks to capture SO_2 and NO_X emissions before they reached the atmosphere, or 3) purchase allowances to cover their emissions quantities.[16] In his PhD dissertation, Herbelot (1992) calculated the value of the option to partake in each of the above-mentioned compliance strategies via a lattice method (effectively an explicit approach for solving the underlying partial differential equations). He calculated the expected cost of SO_2 compliance given these irreversible investment options and the ability to purchase emissions permits, and found the critical SO_2 prices that induce the optimal investment in each of these abatement options. He found the value of the option to invest in retrofitting units to accommodate low-sulphur fuel to be quite low, prompting Insley (2003) to focus only on the option to invest in a scrubber in order to comply with the Title IV standards.

Insley (2003) addressed the optimal decision of an electricity generating firm facing the option to invest in a scrubber or to purchase emissions allowances in order to comply with the

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[15] Ishii and Yan (2004) note that the option value of uncertain regulatory policy is only one of two effects being measured in their Tobit analysis, the other being the impact of restructuring on the costs and revenues of new generation.

[16] Söderholm (1998) and Tuthill (2007, 2008) show that interfuel substitution was a 4th option.

CAAA 1990. She treated the decision as a real options problem, and modelled permit prices as evolving according to a geometric Brownian motion, and used the dynamic programming method to reach her conclusions. Her major contribution was the examination of imbedded options through the formulation of a numerical approach to the investment under uncertainty model involving "time-to-build" (associated with the scrubber installation) and the option to halt the project at any time prior to or after completion.

Blyth et al. (2007) describe the results of the International Energy Agency's (IEA) real-options-based model of decision-making under uncertainty. The model appears quite similar to the ones below, though their perspective is one of industry description. Their objective was to quantify the risk associated with uncertain climate change policy, and to promote real options analysis as tool for firms to use when assessing such investment risks. They represented uncertainty as a potential "step change" (positive or negative) in carbon prices at a fixed future date, and addressed the question of what level of additional investment incentives would be required to overcome the effect of uncertainty on various technologies. They found that the closer firms are in time to the policy change, the greater the policy risk and its effects on investment, and the larger the payoffs (assumed to be increased electricity prices) required to justify immediate investment. Their recommendations, i.e. that increased policy certainty can reduce investment delay in electricity generation, are in line with those below.

A few other papers are worth noting briefly. Rothwell (2006) used a real options approach to evaluate nuclear power plants, and found that uncertainty pertaining to cost, revenues and output all increased the risk-premiums required for investment in a new nuclear facility in Texas. Kiriyama and Suzuki (2004) analysed emissions-free generating investments in Japan and addressed the effects of carbon price uncertainty with a real options model. They considered the invest/don't invest decision for various technologies, and found that increased emissions price uncertainty would lead to higher clean-investment-inducing emissions prices. Finally, Laurikka and Koljonen (2006) found that uncertainty regarding allocation of allowances, as well as allowance price uncertainty, was critical to the decision of switching to natural gas capacity in Finland under the EU ETS, while Laurikka (2006) found in a related study NPV methods were inappropriate in two case studies related to gas-turbine investment in Finland under uncertain CO_2 prices.

Thus, while the existing literature has addressed regulatory or tax policy uncertainty, has modelled the choice between abatement options under GBM allowance price uncertainty, focused on specific investment case studies, or addressed the question of investment timing from an invest/don't invest perspective, it has not, other than Blyth et al. (2007) addressed the impact of future emissions price uncertainty on generating technology choice, and no papers, to my knowledge, have addressed the issue from the US perspective. By modelling CO_2 permit prices as evolving according to two separate stochastic processes, it is possible, as noted, to represent two alternative policy specifications. The first, represented by GBM, assumes government sets and emissions cap and issues allowances which are then traded freely between firms. The second assumes the government can periodically adjust the CO_2 cap, causing permit prices to jump. This case is represented by a combined GBM and Poisson jump (or "jump-diffusion") process. In addition to addressing the impact of CO_2 policy on investment, the present work differs from most previous work by focusing on the plant-type and timing aspects of the investment decision, and not on the when/whether aspects of a given investment decision. The free-boundary problem in this paper is not one separating the "invest" region from the "don't invest" region, but rather the "build dirty" from the "build clean" regions. The objective is to create a methodology for the analysis of the effects of uncertain CO_2 policy (or lack thereof) on the type and timing of capital investment in the electricity generating industry.

3 The Models

The two models presented in this section investigate the optimal plant-type construction decision of an electricity generating firm facing uncertain future CO_2 emissions prices. In what follows, it is assumed that the firm would like to build exactly one generating plant of capacity *G* (either as replacement of a retiring unit or expansion of current capacity) before a known future date, and must now decide whether this plant should be "dirty" or "clean".[17] In order to focus on the effects of uncertain environmental policy, both input and output prices are assumed known and constant, and tradable CO_2 permits are included in the production process to account for environmental policy (and therefore operating cost) uncertainty in the investment decision. As noted[18], "dirty" plants are generally more profitable than "clean" plants in the absence of environmental policy, and "clean" plants become more profitable at higher emissions prices. Thus, at low permit prices, "dirty" plant construction remains optimal, and the objective is to find the critical permit price at which the firm's optimal decision switches from "build dirty" to "build clean".

Rabl and Spadro (2000) used the carbon emissions ratios of the major fossil fuels, and ranked the damage costs of electricity generation by each fuel. They showed that coal-fired generation has the highest damage cost, followed by oil, then natural gas. As such, the "dirty" choice is later defined as a coal-fired combustion unit that produces a large quantity of CO_2 emissions, and the "clean" alternative as a natural gas fired integrated gasification combined cycle (IGCC) unit, though these could, indeed, be any two combustion technologies where the "dirty" technology emits more than the "clean" technology.

In what follows, future plant operating costs are determined by emissions prices and any excess value of a clean plant is assumed to be due to its smaller CO_2 permit requirement. The firm's plant investment is both postponable (thereby introducing the possibility for concern over security of the electricity supply) and irreversible, implying that the orthodox net present value (NPV) theory will not provide a legitimate optimal investment rule. Thus, the opportunity to invest in a clean generating unit rather than the default dirty one is regarded as a financial call option. The option to invest in a clean power plant gleans its value from the same source as a financial call option- from the uncertainty over the future excess value of the clean plant over the dirty one, which, here, depends only on the CO_2 permit price.[19]

Thus, the firm's decision is modelled as an optimal stopping, or free boundary problem. The continuation region in the models below is the region in which, upon investment, it remains optimal for the firm to choose to construct a dirty plant. Because emissions prices are, by assumption, not fixed, permit prices may rise to the point where clean plant construction becomes optimal, bringing the electricity firm's decision into the stopping region. The goal of these models, then, is to find the free boundary of this investment problem, and in so

[17] We therefore assume that the output requirement is either determined exogenously or in a previous decision round, and that the firm now faces the plant-type decision. The clean/dirty decision separation may not be a perfect representation of reality, as different plant-types often serve different load purposes (i.e. base load vs. peak load), but it allows insight into the effects of emission policy uncertainty.

[18] See footnote 3.

[19] See Black and Scholes (1973) or Wilmott et al. (1993) for a full description of option pricing methods.

doing, to determine the effects of the lack of coherent and credible emissions policy on electricity plant-type investment decisions.

The assumptions for the two models are identical, except for those related to the stochastic process for allowance prices. In the first model, I assume permit prices follow a geometric Brownian motion (GBM) with a constant drift rate α. In the second model, CO_2 permit prices follow a combined geometric Brownian motion and Poisson jump process (or a "jump-diffusion" process), which allows permit prices to "jump" as the regulator reduces or increases the emissions cap. Between jumps, however, permits are assumed to continue being traded as financial assets, during which time the price is assumed to follow a geometric Brownian motion.

The assumptions of the two models, then, are as follows:

- There is one stochastic state variable, τ, the CO_2 permit price measured in dollars per ton of CO_2 emissions.
- The regulator can do nothing to alter the market prices of the permits in the first model. In the second model, however, the regulator can cause jumps in the permit price by suddenly announcing a change in the emissions cap, which would cause a jump in the price of available permits.
- Investment costs, future electricity prices, and fuel and operating costs are known with certainty in both models in order to focus on the uncertainty pertaining to environmental policy.[20]
- The firm would like to construct a new plant of predetermined capacity G, and must choose whether its new unit should be dirty or clean.
- τ begins in the continuation region of the problem where it is optimal to construct a dirty unit, and the objective of the model is to find the critical τ^* at which it becomes optimal for the firm to invest in a clean unit instead. The firm's "default choice" is defined as the decision to construct a new dirty unit.
- The firm begins holding the option to invest and the firm's objective is to make the investment decision that maximizes the value of this option.
- Both models employ dynamic programming as their solution technique.
- New information may arrive over time about future permit prices, but the future value of the cost savings associated with a clean plant are always uncertain due to the stochastic process τ follows.
- The CO_2 permit system is implemented in a manor similar to that of the EU ETS and the SO_2 permit scheme in the US. Firms are allocated a number of permits which they then distribute amongst their different generating facilities. Each plant, then, is allocated x permits, authorizing the emission of x tons of CO_2. If the plant's emissions are greater than (less than) x, the firm can buy (sell) extra permits on the permit market, creating an opportunity cost of emitting even in the case where allocated permits cover actual emissions.

[20] Note that correlations between the CO_2 price and other relevant prices (i.e. output and fuel input prices) are likely. For example, CO_2 prices would likely be passed through to electricity prices, thus providing some insurance against CO_2 price uncertainty. It is also likely that increased CO_2 prices would reduce coal demand and increase gas demand, leading to changes in plant operating costs, and the potential strengthening the effects of uncertain emissions prices on plant-type investment decisions.

The difference between the two models thus lies in the assumption regarding the level of potential policy intervention possible, and is captured by the stochastic processes for the allowance price state variable, τ.

3.1 Model 1: Permit Prices Follow a Geometric Brownian Motion (GBM)

In this model, the policy maker cannot alter permit prices once trading has begun. Because permits are assumed to be traded freely between firms, however, their price behaviour is assumed to mimic that of other financial assets and is modelled as having two components: an upward drift and a random walk. Thus, in Model 1, τ evolves according to the following geometric Brownian motion (GBM)

$$d\tau = \alpha \tau dt + \sigma \tau dz \,, \qquad \textbf{Eq. 1}$$

where α is the drift rate, σ is the variance parameter, and dz is an increment of a Wiener process. I do not claim that allowance prices are represented precisely with a GBM, but I proceed with Eq. 1 as a reasonable model of uncertain prices for a traded asset, allowing us to analyse the effects of uncertain emissions prices on plant-level investment behaviour.

There are three characteristics of the Weiner process that will be useful in the discussion and application of the GBM defined by Eq. 1. First, it satisfies the Markov property, meaning that the probability distribution of future values depends only on the current value of the process, and not on any previous value. Second, the Weiner process has independent increments, meaning that the size of a change in the process in any time period is independent of the size of the change in any other time period. Third, the increments of a Weiner process are normally distributed. Formally, $dz = \varphi_t \sqrt{dt}$, where $\varphi_t \sim N(0,1)$, meaning $\mathrm{E}[dz] = 0$ and $Var[dz] = \mathrm{E}[(dz)^2] = dt$.[21] GBMs by definition, then, model percent changes in the variable in question (here $\frac{d\tau}{\tau}$), implying that changes in τ itself are lognormally distributed. If τ follows the GBM defined by Eq. 1, and if τ is currently at the level τ_0, the expected value of τ at some future time t is

$$\mathrm{E}[\tau(t)] = \tau_0 e^{\alpha t} \,, \qquad \textbf{Eq. 2}$$

where E denotes the expectation operator and α is the drift parameter in τ as defined above. It can also be shown that the variance of the future values of τ can be written as

$$Var[\tau(t)] = \tau_0^2 e^{2\alpha t} (e^{\sigma^2 t} - 1) \,, \qquad \textbf{Eq. 3}$$

which will be useful in interpreting the results of the model.[22]

The model below uses this GBM for τ and ascertains the firm's optimal plant-type investment choice under uncertain future emissions costs in three steps. Step 1 involves the calculation

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[21] See Dixit and Pindyck (1994) and Cox and Miller (1965) for a more detailed discussion of Weiner processes and GBMs.
[22] See Dixit (1993), Dixit and Pindyck (1994), and Aitchinson and Brown (1957) for more on the characteristics of GBMs and the lognormal distribution.

of $\theta(\tau)$, the excess payoff the firm can expect to receive if it invests in the clean plant rather than the default dirty one. In Step 2, the differential equation for $F(\tau)$, the value of the option to invest in the clean plant, is found through the dynamic programming approach and the boundary conditions for its solution are presented. Finally, in Step 3, the differential equation is solved, giving the value of the clean plant investment option and the critical τ^* at which it becomes optimal to invest in the clean plant.

STEP 1:

The value of the clean investment is defined to be the excess annual payoff that a firm receives from building a clean plant rather than a dirty one, which is just the difference between the profits earned by a clean plant and those earned by a dirty one ($\Pi_C - \Pi_D$).[23] The annual profit earned by a dirty or clean plant, Π_D and Π_C, respectively is defined as

$$\Pi_D = (p - c_D)q - (M - x)\tau \qquad \textbf{Eq. 4}$$

and

$$\Pi_C = (p - c_C)q - [(1-\phi)M - x]\tau\ , \qquad \textbf{Eq. 5}$$

where p is the price per MWh of electricity, q is the number of MWh generated by the plant in a year, and c_D and c_C are the non-emissions variable costs (i.e. fuel and operation and maintenance) per MWh for the dirty and clean plant respectively. In general, $c_D < c_C$. (EIA, 2004a, 2004b). M is the fixed quantity of emissions produced by a plant using the dirty technology measured in tons/year, and ϕ is the percentage of the dirty plant's emissions saved by constructing a clean unit. x represents the number of permits allocated to the plant, where each permit allows the firm one ton of CO_2 emissions. The second term on the right hand sides of both equations 4 and 5 can be either positive or negative, depending on whether the firm's generating technology requires it to purchase or allows it to sell extra permits.

The payoff per year from building a clean plant in lieu of a dirty one, $\Pi_C - \Pi_D$, is then

$$\Pi_C - \Pi_D = \{(p - c_C)q - [(1-\phi)M - x]\tau\} - \{(p - c_D)q - (M - x)\tau\}$$
$$\Pi_C - \Pi_D = (c_D - c_C)q + \tau M\phi = [\Delta c]q + \tau M\phi\ . \qquad \textbf{Eq. 6}$$

Here, $\Delta c = c_D - c_C$, represents the excess variable cost of generation required to produce electricity with the clean technology. Recall, however, that $c_D < c_C$, meaning Δc has a negative value. The second term on the right hand side of Eq. 6 represents the cost savings associated with the reduced emissions, and

[23] Recall that prices and quantities for other inputs and outputs are fixed and known.

I assume for simplicity that either plant, once constructed, is infinitely-lived[24], meaning that the present value of the future cost savings associated with operating a clean plant rather than a dirty one looks as follows:

$$\theta(\tau) = \int_0^\infty \left([\Delta c]q + E[\tau(t)]M\phi\right)e^{-\rho t}dt\,, \qquad \textbf{Eq. 7}$$

where $E[\tau]$ denotes the expected value of τ, and ρ is the firm's discount rate. Recalling the expected value of τ from Eq. 2,

$$\theta(\tau) = \int_0^\infty ([\Delta c]q + \tau_0 e^{\alpha t}M\phi)e^{-\rho t}dt$$

$$\theta(\tau) = \int_0^\infty [\Delta c]qe^{-\rho t}dt + \int_0^\infty \tau_0 M\phi e^{(\alpha-\rho)t}dt\,,$$

or simply

$$\theta(\tau) = \frac{[\Delta c]q}{\rho} + \frac{\tau_0 M\phi}{\rho - \alpha} \qquad \textbf{Eq. 8}$$

where τ_0 is the current value of τ. I will assume that the firm is risk neutral so that the firm's discount rate is equal to the risk free interest rate (i.e. $\rho = r$).[25] Thus, equation 8 becomes

$$\theta(\tau) = \frac{[\Delta c]q}{r} + \frac{\tau_0 M\phi}{\delta} \qquad \textbf{Eq. 9}$$

where I have followed Dixit and Pindyck (1994) in denoting the difference between r and α by δ, such that $\delta \equiv r - \alpha$. Here, δ effectively represents the appropriate discount rate, adjusted for τ's drift rate, to be applied to the uncertain portion of the present value of the clean plant's excess future profits.

STEP 2:

The optimal investment rule maximizes the value of the firm's investment option, and requires the comparison of the present value of immediate investment in the clean plant with that of waiting and holding the option. Recall that this problem has been defined as an

[24] We argue that it is reasonable to ignore plant life as an explicit variable given Herbelot's (1992) findings that critical investment values were not significantly impacted by different plant life assumptions and Ellerman's (1998) findings on plant life extension.

[25] Risk neutrality is assumed in order to focus on the effects of allowance price volatility, to which the investment decision is sensitive regardless of the firm's risk preferences. (Dixit and Pindyck (1994)). Note that risk aversion would lead to a lower $\theta(\tau)$ and a larger critical τ^*. (See, e.g., Hugonnier and Morellec (2007).)

optimal stopping problem, and that the firm is assumed to begin in the continuation region, where $\tau < \tau^*$, and the dirty plant remains optimal.[26]

The firm's objective in each period is to maximize the sum of current profit flow from not exercising the option (zero in this case) and the discounted continuation value, defined as the value of having the option in the next period. Thus, the firm is assumed to maximize its return from holding the option, which is just equal to the option's capital appreciation. If $F(\tau)$ represents the value of the option to invest in a clean plant instead of a dirty one, then the firm will make the exercise decision such that

$$F_t(\tau_t) = \max\left\{\frac{1}{1+r} E_t[F_{t+1}(\tau_{t+1})]\right\}. \qquad \textbf{Eq. 10}$$

In continuous time, the Bellman equation for this problem in continuous time looks as follows:[27]

$$rF(\tau) = \frac{1}{dt} E[dF(\tau)]. \qquad \textbf{Eq. 11}$$

This Bellman equation yields the following second order differential equation in $F(\tau)$, which will be solved for the value of the option to invest in a clean plant and used to calculate the critical τ^*:[28]

$$0 = \frac{1}{2}\sigma^2\tau^2 F''(\tau) + (r-\delta)\tau F'(\tau) - rF(\tau), \qquad \textbf{Eq. 12}$$

where I have replaced α with r-δ in order to eliminate one parameter from the calculations.

Equation 12 must be solved subject to the following boundary conditions:

$$F(0) = 0 \qquad \textbf{Eq. 13}$$

$$F(\tau^*) = \theta(\tau^*) - \Delta I \qquad \textbf{Eq. 14}$$

$$F'(\tau^*) = \theta'(\tau^*) \qquad \textbf{Eq. 15}$$

In Eq. 14, $\Delta I = I_C - I_D$, where I_C and I_D are the sunk investment costs required for a clean and dirty unit, respectively. Thus, ΔI is the additional expenditure required for the construction of a clean generating unit, and is measured in dollars.

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[26] In order for the continuation region to be below τ^* rather than above, it must be true that the sum of the current payoff from holding the option (in this case, zero) and the discounted continuation value, $F(\tau)$, must be decreasing as τ increases. i.e. we require that $\frac{d}{d\tau}(-F(\tau)) < 0$, which does in fact hold in these models.

[27] See Appendix 1 for the derivation.

[28] See Appendix 2 for the derivation of differential equation 12.

Eq. 13 is based on the fact that if τ ever goes to zero, it will remain at zero forever. This is a characteristic of the lognormal distribution and is implied by τ's GBM process. If the permit price ever falls to zero, then, the option to invest in a clean generating unit will have no value. Equations 14 and 15 consider the optimal investment decision, where τ^* is the critical permit price at which clean plant investment becomes optimal, and marks the free boundary between the continuation and stopping regions. Eq. 14, the value-matching condition, ensures that $F(\tau)$ is continuous at the critical clean investment permit price τ^* and simply states that at τ^*, the value of holding the investment option exactly equals the net investment payoff $\theta(\tau^*) - \Delta I$. Eq. 15 is the smooth-pasting condition, in which the primes indicate derivatives with respect to τ. Eq. 15 ensures that the slopes of $F(\tau)$ and $\theta(\tau)$ are equivalent at τ^*. Together, equations 14 and 15 guarantee that the firm could not do better by deciding to invest in clean generating technologies at a different τ. (See Dixit (1993) and Dixit and Pindyck (1994).)

STEP 3:

Because Eq. 12 is linear in $F(\tau)$ and its derivatives, the general solution to 12 will be a linear combination of any two independent solutions, $F(\tau) = A\tau^{\beta}$, and will thus be of the form $F(\tau) = A_1\tau^{\beta_1} + A_2\tau^{\beta_2}$, where A_1 and A_2 are constants to be determined, and β_1 and β_2 are the roots of the characteristic equation (Dixit and Pindyck, 1994).[29] Boundary condition 13 implies that if $\tau \to 0, F(\tau) \to 0$, and I show in Appendix 3 that $\beta_1 > 0$ and $\beta_2 < 0$. Thus, $A_2=0$ and the solution form is

$$F(\tau) = A_1\tau^{\beta_1}. \qquad \textbf{Eq. 16}$$

The equation for τ^* results from plugging Eq. 16 into boundary conditions 14 and 15 and some algebraic manipulation:

$$\tau^* = \frac{\beta_1}{(\beta_1 - 1)} \frac{\delta}{M\phi} \left[\Delta I - \frac{[\Delta c]q}{r} \right]. \qquad \textbf{Eq. 17}$$

Thus, the clean-investment-inducing critical τ^* is decreasing in β_1 (which depends upon σ, δ and r) and Δc. Recall from above, however, that $\Delta c = c_D - c_C$ and has a negative value. Thus, as might be expected, τ^* is increasing with the additional costs (both investment and variable) incurred through constructing and operating a clean plant rather than a dirty one.

Equation 17 also shows that τ^* shares an inverse relationship with M (the emissions factor for the dirty technology) and with ϕ (the efficiency rate, or proportion of emissions reduced when a clean unit is selected). This should be expected, as the more technically efficient (i.e. less polluting) the clean generating option is, the smaller will be unit's required number of carbon permits. Thus more efficient clean technology options imply a larger cost saving for the firm, making the clean investment optimal at a lower permit price.

[29] See Appendix 3 for the derivation of the characteristic equation and its solutions.

Plugging Eq. 17 into Eq. 9 yields the equation for $\theta(\tau^*)$:

$$\theta(\tau^*) = \frac{1}{\beta_1 - 1}\frac{[\Delta c]q}{r} + \frac{\beta_1}{\beta_1 - 1}\Delta I \;=\; \frac{1}{\beta_1 - 1}\left[\frac{[\Delta c]q}{r} + \beta_1 \Delta I\right]. \qquad \textbf{Eq. 18}$$

Inserting Eq. 16 and Eq. 18 into the value-matching boundary condition (Eq. 14), rearranging and simplifying, yields the equation for A_1:

$$A_1 \;=\; \frac{\theta(\tau^*) - \Delta I}{\tau^{*\beta_1}} \;=\; \frac{\left(\dfrac{1}{\beta_1 - 1}\right)\left[\Delta I - \dfrac{(\Delta c)q}{r}\right]}{\left\{\left(\dfrac{\beta_1}{\beta_1 - 1}\right)\dfrac{\delta}{M\phi}\left[\Delta I - \dfrac{(\Delta c)q}{r}\right]\right\}^{\beta_1}}. \qquad \textbf{Eq. 19}$$

I present Model 2 before further discussing the characteristics of both models' solutions in Sections 4 and 5.

3.2 Model 2: Permit Prices Follow a Combined Geometric Brownian Motion and Poisson Jump Process

In this model, τ is assumed to follow a combined GBM and Poisson jump process. Here, regulators can affect the price of permits (and, therefore, the value of clean and dirty capital investments) by suddenly changing the emissions cap. It is assumed that firms have no prior knowledge of when these price jumps may occur. Policy changes, then, are "events" of a Poisson process, and λ represents the mean arrival rate of an event during the interval *dt*. The probability of a policy change during any short period of time *dt* is therefore λdt, and the probability of no change is $1- \lambda dt$. Changes in emissions policy are assumed to cause permit prices to jump by some percentage *u* of their current level.[30] Thus $-1 \leq u$, where a positive *u* implies the tightening of environmental policy and a rise in the permit price, while a negative *u* implies a loosening of environmental policy and a drop in permit prices. Between jumps, permits are freely traded between firms, and prices are assumed to evolve according to a GBM as described in Model 1.

Here, *q* represents a Poisson process analogous to that of the Wiener process in Model 1, with an event being defined as a jump of size *u*, where. Thus,

$$dq = \begin{cases} 0 & \text{with probability } 1 - \lambda\text{dt} \\ \text{u} & \text{with probability } \lambda\text{dt} \end{cases}$$

and the stochastic process that τ follows is

$$d\tau = \alpha_\tau \tau dt + \sigma_\tau \tau dz + \tau dq \qquad \textbf{Eq. 20}$$

[30] Here, *u* is a parameter, though it is possible to generalize the model to the situation where *u* itself is a random variable.

where α_τ, σ_τ, and dz have the same interpretations as before, and dq is assumed independent of dz.

A few important characteristics of the permit price process 20 should be noted. First, as shown by Merton (1976), a variable evolving according a jump-diffusion process has the following expected value:[31]

$$E[\tau] = \tau_0 \exp\left[\alpha + \lambda u\right], \qquad \textbf{Eq. 21}$$

where τ_0 represents the current value of τ. Also, α no longer represents the expected percentage rate of change in τ as it did in Model 1. Here,

$$\frac{1}{dt}\frac{E(d\tau)}{\tau} = \alpha + \lambda u. \qquad \textbf{Eq. 22}$$

Following Dixit and Pindyck (1994), assume for the moment that $\alpha=0$ in order to find the variance of $d\tau$. There are then three possibilities for the change in τ in any infinitesimal period dt, and they are as follows:

$$d\tau = \begin{cases} \sigma\tau\sqrt{dt} & with\ probability \frac{1}{2}(1-\lambda dt) \\ -\sigma\tau\sqrt{dt} & with\ probability \frac{1}{2}(1-\lambda dt) \\ u\tau & wtih\ probability\ \lambda dt \end{cases} . \qquad \textbf{Eq. 23}$$

Thus, we have

$$E[d\tau] = \frac{1}{2}(1-\lambda dt)\sigma\tau\sqrt{dt} - \frac{1}{2}(1-\lambda dt)\sigma\tau\sqrt{dt} + u\tau\lambda dt = u\tau\lambda dt \qquad \textbf{Eq. 24}$$

$$(d\tau)^2 = \sigma^2\tau^2 dt + u^2\tau^2 \qquad \textbf{Eq. 25}$$

$$E[(d\tau)^2] = (1-\lambda dt)\sigma^2\tau^2 dt + \lambda dt u^2\tau^2 \qquad \textbf{Eq. 26}$$

$$[E(d\tau)]^2 = u^2\tau^2\lambda^2 dt^2 \qquad \textbf{Eq. 27}$$

and finally,

$$Var[d\tau] = E[(d\tau)^2] - [E(d\tau)]^2$$

$$Var\left[d\tau\right] = \sigma^2\tau^2 dt + \lambda u^2\tau^2 dt. \qquad \textbf{Eq. 28}$$

[31] See also Dixit (1993) and Aitchinson and Brown (1957) for a discussion of the lognormal distribution.

where terms in dt^2 have been eliminated because they tend to zero faster than terms in dt as $dt \rightarrow 0$. Eq. 28 will be important for the interpretations in the next section.

The goal of the firm is still to maximize the discounted expected future value of the plant it decides to build, and still must choose between building a dirty plant or a clean one. The steps involved in the solution process are the same.

STEP 1:

Just as above, the excess value of a clean plant is:

$$\theta(\tau) = \int_0^\infty \left([\Delta c]q + E[\tau(t)]M\phi\right) e^{-rt} dt. \qquad \textbf{Eq. 29}$$

Using the expectation of τ from Eq. 21,

$$\theta(\tau) = \frac{[\Delta c]q}{r} + \frac{\tau_0 M\phi}{\delta - \lambda u}, \qquad \textbf{Eq. 30}$$

where, as above, τ_0 is the current value of τ.

STEP 2:

The firm's objective is still

$$F(\tau) = \max\left(\frac{1}{1 + r\Delta t}\right) E[F(\tau')],$$

and the Bellman equation for the problem has not changed, though it is repeated here for convenience:

$$rF(\tau) = \frac{1}{dt} E[dF(\tau)]. \qquad \textbf{Eq. 31}$$

Because of the stochastic process τ follows in this model, a different version of Ito's lemma is required in the expansion of the right hand side of Eq. 31, yielding the following differential equation for $F(\tau)$:

$$0 = \frac{1}{2}\sigma^2\tau^2 F''(\tau) + \alpha\tau F'(\tau) + \lambda F[(1+u)\tau] - (\lambda + r)F(\tau).$$ [32] **Eq. 32**

Equation 32 must be solved according to the same boundary conditions as above and for the same reasons. These are:

[32] See Appendix 4 for the derivation.

$F(0) = 0$ **Eq. 33**

$F(\tau^*) = \theta(\tau^*) - \Delta I$ **Eq. 34**

$F'(\tau^*) = \theta'(\tau^*)$ **Eq. 35**

STEP 3:

As Eq. 32 is again linear in $F(\tau)$ and its derivatives $F'(\tau)$ and $F''(\tau)$, its solution will again be a linear combination of its independent solutions, $F(\tau) = A_1\tau^{\beta_1} + A_2\tau^{\beta_2}$. (See Dixit and Pindyck (1994).) β_1 and β_2 will have the same signs (though not necessarily the same magnitude) as they did in Model 1, and boundary condition 33 again implies that A_2=0, meaning the solution to Eq. 32 has exactly the same form as it did in Model 1:

$F(\tau) = A_1\tau^{\beta_1}$. **Eq. 36**

The difference lies in the fact that β_1 now solves a different characteristic equation, namely:[33]

$$\frac{1}{2}\sigma_\tau^2\beta_1(\beta_1-1)+(r-\delta)\beta_1+\lambda(1+u)^{\beta_1}-(\lambda+r)=0, \qquad \textbf{Eq. 37}$$

where, again, I have replaced α with r-δ. The solution for β_1 from Eq. 37 can not be found analytically and must therefore be solved numerically. Examples are provided in the next section.

Given the solution form for $F(\tau)$, $F(\tau) = A_1\tau^{\beta_1}$, however, the critical τ^* can be calculated as above. The steps, starting with equation 16, are exactly the same, and they are omitted here to save space. The solution for the critical τ^* is

$$\tau^* = \frac{\beta_1}{(\beta_1-1)}\left[\frac{\left(\Delta I - \frac{[\Delta c]q}{r}\right)*(\delta-\lambda u)}{M\phi}\right]. \qquad \textbf{Eq. 38}$$

Note that its form is similar to that of the first model, but that its value will differ not only through the effects of λ, but also through of the new solution for β_1, which depends on σ, δ, λ, u and r. The comparative statics for the parameters ΔI, r, q, M, φ, δ, σ, and Δc in Eq. 38 are the same as those for Eq. 17, and are not repeated here. λ and u, however, are new, and note that $\frac{\partial \tau^*}{\partial \lambda} < 0$ and $\frac{\partial \tau^*}{\partial u} < 0$. Recall that u can be either positive or negative, meaning $\frac{\partial \tau^*}{\partial u} < 0$ implies that larger negative jumps in CO_2 permit prices lead to a higher critical τ^*, while larger positive jumps in the permit price imply a lower critical τ^*. These issues and

[33] See Appendix 2.5 for the derivation.

the differences in the predictions of GBM and the jump-diffusion models are discussed in more detail in the following section.

The solution process for A_1 in Eq. 36 is also the same, and when τ follows a jump-diffusion process, A_1 looks as follows:

$$A_1 = \frac{\left(\frac{1}{\beta_1 - 1}\right)\left[\Delta I - \frac{(\Delta c)q}{r}\right]}{\left\{\frac{\beta_1}{(\beta_1 - 1)}\left[\frac{\left(\Delta I - \frac{[\Delta c]q}{r}\right)*\left(\delta - \lambda u\right)}{M\phi}\right]\right\}^{\beta_1}}. \qquad \textbf{Eq. 39}$$

4 Examples and Analysis

This section provides examples that allow for the examination of the characteristics of the investment rules found through the models presented above. In what follows, I consider a specific investment decision in which the firm faces two distinct unit-type alternatives, and I calculate the critical τ^* and the value of the investment option.[34] costs per kWh and the, the results hold more generally. The following section will show the dependence of the solutions on the parameter values.

In what follows, the risk-free interest rate, r, is set equal to 0.05, which is approximately equal to the average yield of 4.91% on 20-year US Treasury bills in 2007 (Federal Reserve), and α is assumed to equal zero so there is no drift in CO_2 permit prices in the base case examples that follow, and δ=0.05.[35] It is assumed that the volatility characteristics of the market for CO_2 permits would be similar to those of the market for SO_2 permits and I proxy for σ, the variance parameter in the equations for the evolution of τ, with implied volatility. Thus, following Insley (2003), σ =0.3.

The remainder of the parameter assumptions are plant-specific. It is assumed that the firm would like to build a 50 MW unit, and is deciding between a new scrubbed coal unit (the dirty option), and a new natural gas-fired integrated gasification combined cycle (IGCC- the clean option). In order to keep the grid balanced in the case of a supply or demand shock, firms are required to maintain a spinning reserve at all times, meaning that generating units are seldom, if ever, operated at full capacity. Thus, it is assumed that upon completion, the firm would operate its new 50 MW unit at an average capacity factor (usage rate) of 80%, implying that the new unit would provide q= 350,400 MWh/year.[36]

[34] The solutions in this section are example-specific, but hold generally for any two-unit decision where $\left(\Delta I - \frac{[\Delta c]q}{r}\right) > 0$, or $\Delta I > \frac{[\Delta c]q}{r}$. If this is not true, the clean investment is not costly enough relative to the dirty investment to warrant delay. A_1 would then have zero (or negative) value, as would $F(\tau)$, and immediate investment in the clean option would be optimal.

[35] Recall that $\delta \equiv r-\alpha$, where δ represents the discount rate applied to the uncertain portion of the clean unit's excess value.

[36] q= 50 MW * 80% of full capacity * 8,760 hours per year.

In order to find *M*, the quantity of annual CO_2 emissions from a new scrubbed coal unit, I take the annual heat input requirements (measured in mmBtu) for a new 50 MW scrubbed coal unit with an average capacity factor of 80%, and multiply by the average number of pounds of CO_2 associated with each mmBtu provided by coal combustion. According to the EIA (EIA, 2004b), a new scrubbed coal unit would require 9.0 mmBtu for each MWh of electricity, meaning that the new scrubbed coal unit would require 3,153,600 mmBtu of thermal heat input per year. According to the EPA (1999), coal combustion creates an average of 212.7 pounds of CO_2 per mmBtu of heat energy, meaning that the new scrubbed coal unit would emit approximately 670,770,720 pounds, or 335,385 tons of CO_2 each year. A new 50 MW natural gas-fired IGCC unit (also operated at an average capacity rate of 80%) would emit only 152,612 tons of CO_2 each year using similar calculations[37], or 54.5% less than the coal plant. Thus, in what follows, *M*= 335,385 tons of CO_2 per year, and ϕ=0.545.

Using data on operation and maintenance (O&M) costs (EIA, 2004b, Table 38) and information on the cost per unit of heat energy of coal and natural gas (EIA, 2002, Table 4.5), the variable cost per MWh of electricity from a new scrubbed coal plant is calculate to be $17.935, and $32.116 from a new natural gas-fired IGCC. The difference in these costs, Δc, is then $c_d - c_c =$-$14.18/MWh. Using data on the overnight construction costs (EIA, 2004b, Table 38), I_D for a 50 MW scrubbed coal unit would be $54,550,000, while I_C for a 50 MW natural gas-fired IGCC would be $25,800,000. Thus, $\Delta I = I_c - I_d =$-$28,750,000.

For Model 2, two additional assumptions must be made: one regarding the probability of a jump in permit prices in an infinitesimal period *dt*, and one about the size of such a jump, should it take place. I assume $\lambda = 0.1$ and *u* is equal to either +0.2 or -0.2 (depending on expectations about future regulatory stringency), meaning that the probability of a jump occurring during time *dt* is 0.1*dt*, and should the jump occur, it will change permit prices by positive or negative twenty percent. All parameters are summarized in Table 1 below.

Table 1: Parameter Values

Parameter	Value
r	0.05
δ	0.05
σ	0.3
q	350,400 MWh/year
M	335,385 tons CO_2/ year
ϕ	0.545
Δc	-$14.18/MWh
ΔI	-$28,750,000
λ	0.1
u	0.2

[37] The IGCC would require 8,000 Btu/kWh with an emissions rate of 117.6 lb/mmBtu (EIA (2004b) and EPA (1999).

These parameter assumptions are used in the models described in the previous section to analyse the optimal investment decision of an electricity generating firm faced with a new-scrubbed-coal vs. IGCC construction decision under the two stochastic processes for permit price evolution. For the sake of comparison, I begin by analyzing the problem with the NPV methodology assuming no uncertainty over permit prices. Though the context of this example is quite specific, both models can be generalized to incorporate different unit sizes as well as different unit-type alternatives and other assumptions related to the stochastic processes.

4.1 Stationary Deterministic Case

As a benchmark, I begin with the stationary deterministic case where there is no uncertainty over future permit prices. This case could represent that of a known and certain CO_2 tax, as compared to a cap-and-trade emissions scheme.[38] Here, $\alpha=\sigma=\lambda=0$, meaning that the current CO_2 price, τ, will remain unchanged forever. Because the revenues ($p*q$) are identical for both plants, the critical τ^* will be the permit price at which the present value of future costs associated with the two units are equal. The present value of the construction and operating costs of a new scrubbed coal unit with permit price τ is

$$C_d = \int_0^\infty (17.935 \times 350{,}400) e^{-0.05t} dt + \int_0^\infty (\tau \times 335{,}385) e^{-0.05t} dt + 54{,}500{,}000$$

or

$$C_d = \frac{17.935 \times 350{,}400}{0.05} + \frac{\tau \times 335{,}385}{0.05} + 54{,}500{,}000. \qquad \textbf{Eq. 40}$$

The same calculation for the new natural gas-fired IGCC unit yields

$$C_c = \int_0^\infty (32.116 \times 350{,}400) e^{-0.05t} dt + \int_0^\infty (\tau \times 335{,}385 \times 0.545) e^{-0.05t} dt + 25{,}800{,}000$$

or

$$C_c = \frac{32.116 \times 350{,}400}{0.05} + \frac{\tau \times 335{,}385 \times 0.545}{0.05} + 25{,}800{,}000. \qquad \textbf{Eq. 41}$$

Equating the right-hand sides of equations 40 and 41, $\tau^*= 23.16$ when permit prices are known with certainty. At this level, a generating firm is indifferent between constructing a new coal unit or a new IGCC unit. If $\tau< \tau^*$at the time the construction decision is made, construction of a new scrubbed coal unit remains optimal, whereas if $\tau> \tau^*$, construction of the IGCC unit is optimal.

[38] Note that a model of emissions taxes allowing the regulator to ratchet the tax level up or down could also be constructed. This would involve modeling the permit price as a Poisson process (leaving out the GBM evolution between jumps), and could be accomplished by setting $\alpha=\sigma=0$ in the Poisson-GBM model.

4.2 Model 1: τ follows a GBM

In Model 1, CO_2 prices evolve according to the following geometric Brownian motion:

$$d\tau = \alpha\tau dt + \sigma\tau dz . \qquad \textbf{Eq. 42}$$

Recall from the previous section that in Model 1

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \frac{\sqrt{(r-\delta)^2 + (\delta - r + \frac{1}{4}\sigma^2 + 2r)\sigma^2}}{\sigma^2} > 1, \qquad \textbf{Eq. 43}$$

which, with the above parameter assumptions, yields $\beta_1 = 1.\overline{6}$. Using this value in the equation for the critical permit price,

$$\tau^* = \frac{\beta_1}{(\beta_1 - 1)}\frac{\delta}{M\phi}\left[\Delta I - \frac{[\Delta c]q}{r}\right] \qquad \textbf{Eq. 44}$$

τ^* = \$48.30 per ton of CO_2, meaning that when future CO_2 prices are modelled by equation 42 a generating firm would find it optimal to build a new scrubbed coal unit rather than the cleaner natural gas-fired IGCC until permit prices reach \$48.30/ton of CO_2. When τ>\$48.30, it becomes optimal for the firm to build a natural gas-fired IGCC instead. Thus, the clean-investment-inducing τ^* here is more than twice that when τ is known with certainty.

When the assumptions hold, the value of A_1,

$$A_1 = \frac{\left(\frac{1}{\beta_1 - 1}\right)\left[\Delta I - \frac{(\Delta c)q}{r}\right]}{\left\{\left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{\delta}{M\phi}\left[\Delta I - \frac{(\Delta c)q}{r}\right]\right\}^{\beta_1}} \qquad \textbf{Eq. 45}$$

is 165,372, meaning that the value of the option to invest in the clean unit instead of the dirty one, $F(\tau) = A_1\tau^{\beta_1}$, is

$$F(\tau) = 165{,}372\tau^{1.\overline{6}}. \qquad \textbf{Eq. 46}$$

4.3 Model 2: τ Follows a Jump-Diffusion Process

If policy makers have the ability to interfere with the emissions cap once a cap-and-trade scheme has been established, the stochastic process that permit prices follow is more realistically represented by a mixed Poisson-GBM process, yielding quite different results. Recall that in Model 2 permit prices evolve according to

$$d\tau = \alpha\tau dt + \sigma\tau dz + \tau dq \quad \textbf{Eq. 47}$$

Recall also that in the jump-diffusion model, β_1 must be found numerically as the solution to[39]

$$\frac{1}{2}\sigma^2\beta_1(\beta_1 - 1) + (r-\delta)\beta_1 + \lambda(1+u)^{\beta_1} - (\lambda + r) = 0 . \quad \textbf{Eq. 48}$$

Given the above parameter assumptions, and remembering that $\alpha = r - \delta$ by definition, β_1=1.96643 when u is -0.2 and β_1= 1.35829 when u is +0.2.

Using these values in the equation for the critical permit price from Case 2 of the mixed GBM-Poisson model,

$$\tau^* = \frac{\beta_1}{(\beta_1 - 1)}\left[\frac{\left(\Delta I - \frac{[\Delta c]q}{r}\right)*\left(\delta - \lambda u\right)}{M\phi}\right], \quad \textbf{Eq. 49}$$

when u is -0.2, τ^*= \$55.03/ton of CO_2 emissions, 14% larger than in the GBM model. When u is +0.2, τ^*= \$43.94/ton, 9% lower than the GBM model with no Poisson jumps, but still 90% larger than in the NPV case of certain emissions prices. The equation for A_1 from Model 2,

$$A_1 = \frac{\left(\frac{1}{\beta_1 - 1}\right)\left[\Delta I - \frac{(\Delta c)q}{r}\right]}{\left\{\frac{\beta_1}{(\beta_1 - 1)}\left[\frac{\left(\Delta I - \frac{[\Delta c]q}{r}\right)*\left(\delta - \lambda u\right)}{M\phi}\right]\right\}^{\beta_1}}, \quad \textbf{Eq. 50}$$

yields A_1=27,607 and $F(\tau)$ = 27,607 $\tau^{1.96643}$ for u=-0.2 and A_1=1,156,735 and $F(\tau)$ = 1,156,735 $\tau^{1.35829}$ when u=+0.2. Results are summarized in Table 2.

Table 2: Model Solutions Given Above Parameter Assumptions

	Stationary Deterministic case: $\alpha = \sigma = 0$	GBM	Poisson-GBM (negative u)	Poisson-GBM (positive u)
β_1		$1.\overline{6}$	1.96643	1.35829

[39] We have used Mathematica's 'FindRoot' command, which employs a damped Newton's method, as the solution means for the roots of this equation.

τ^* ($/ton CO_2)	$23.16	$48.20	$55.03	$43.94
A_1		165,944	27,607	1,156,735
$F(\tau)$		$165{,}944\tau^{1.\bar{6}}$	$27{,}607\ \tau^{1.96643}$	$1{,}156{,}735\tau^{1.35829}$

It is obvious that uncertain future permit values lead to an increase in the required excess return of the clean unit as determined by τ. As noted, this is due to the fact with uncertain emissions prices, τ may fall below τ^* tomorrow, rendering the clean plant investment decision sub-optimal. As a result, there is an opportunity cost of investing in a clean plant today and this opportunity cost increases with the probability of lower future CO_2 prices, as can be seen most intuitively from the higher τ^* for the negative Poisson jump example. The firm prefers to retain its investment option until τ has reached a level that ensures the optimality of the clean unit's construction. The greater the probability of lower future CO_2 prices, the higher will be the τ^* that satisfies this condition. Assuming that exogenous market forces determine capacity requirements and thereby create deadlines for new capacity or for replacement of retiring capacity, the higher critical τ^*s in the presence of GBM and negative-jump Poisson-GBM uncertainty reduces the probability that permit prices will reach τ^* before the firm's investment decision deadline, implying that firms are more likely to invest in the dirty option when emissions policy is uncertain and weak.

In the context of this example, IGCC construction becomes optimal soonest in the stationary deterministic model when permit prices are stationary and deterministic (i.e. when emissions policy is a known fixed tax), then in Poisson-GBM model with positive jumps, followed by the GBM model, and finally under the Poisson-GBM model with negative jumps. This implies that the dirty plant is most likely to be chosen in the negative jump-diffusion model, when the potential drop in future CO_2 permit prices creates the largest downside risk for investment in the clean plant. Note that the government may prefer to remain uncommitted regarding environmental policy[40] and that without performing a welfare analysis, government objectives should not be prescribed, but if it were found optimal for the government to minimize the effects of emissions price uncertainty on clean capacity investment delay, these models recommend a credible and certain CO_2 tax as the most appropriate policy with the next best option being a cap-and-trade scheme whose cap is either sufficiently high or expected to become tighter.

5 Parameter Dependence and Analysis

The results above are obviously dependent upon the parameter assumptions outlined in Table 1. Note that the calculated τ^* values are not claimed to be perfectly realistic (recall that I have abstracted from price and quantity uncertainty, for example), but their relative values are useful and carry interesting policy implications. The remainder of this section focuses on how the results from the two models depend on the parameter assumptions, particularly those related to the evolution of emissions prices. I will look first at the GBM model, then at the two cases of the Poisson-GBM model, and then compare the two.

[40] See the discussion at the end of the next section and Pindyck (2000).

5.1 Model 1: GBM

The GBM process in Model 1 implies that a lower value of τ is equally likely as a higher value of τ in any period. The chance that τ may fall creates an opportunity cost of immediate investment in the clean plant and the value of the option to invest is equal to this opportunity cost. Thus, assuming the firm has not arrived at its investment deadline, it continues to hold the investment option at low values of τ (i.e. when $\tau<\tau^*$, because in this region $F(\tau) > \theta(\tau) - \Delta I$), and this option has a value of $F(\tau) = A_1\tau^{\beta_1}$. Once τ reaches the critical τ^*, the value of the option and the value of the clean plant become equal, the option is exercised, and the firm receives $\theta(\tau) - \Delta I$. The critical permit price at which exercising the clean-instead-of-dirty option is therefore found at the point where $F(\tau) = A_1\tau^{\beta_1}$ becomes tangent to $\theta(\tau) - \Delta I$ (see Dixit and Pindyck, 1994). Higher levels of uncertainty (as measured by σ) lead to a tangency at higher values of τ. This is made explicit in Figure 1.

Figure 1: *F(τ)* as a Function of τ for Different Values of σ[41]

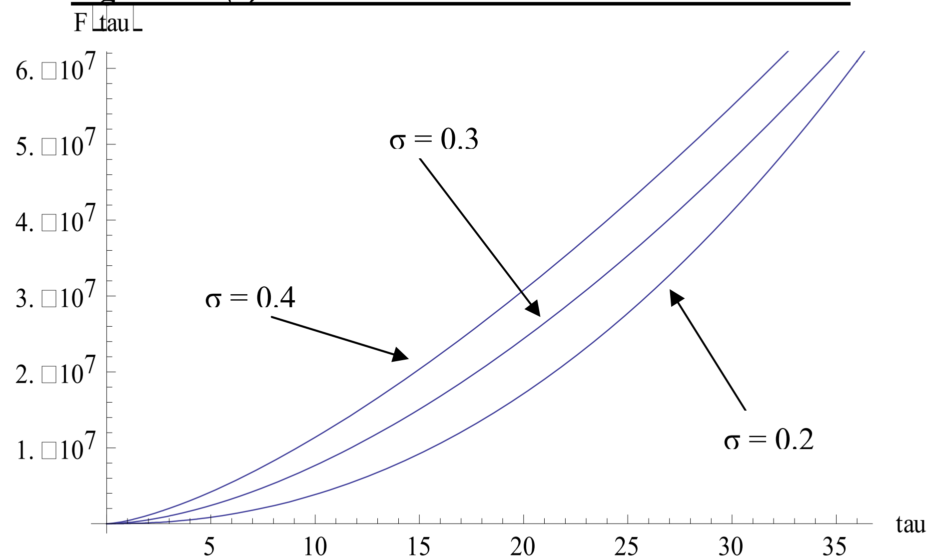


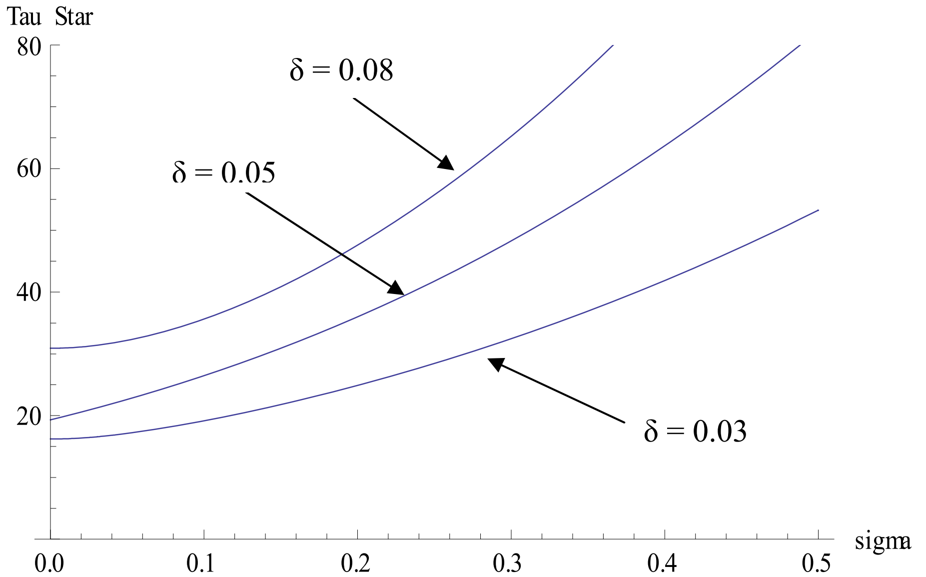
Figure 1 presents the value of the option to invest in the clean IGCC unit rather than the dirty scrubbed coal unit, $F(\tau)$, in the GBM model as a function of the price of CO_2 permits for different values of σ, the variance parameter of the CO_2 permit price process.[42] An increase in σ increases the risk that the firm will learn tomorrow that permit prices have fallen low enough to have made the dirty plant the optimal, meaning that increasing σ leads to a greater opportunity cost of investing in the clean alternative today rather than waiting. As such, increasing the value of σ causes an increase in the value of the option to invest, $F(\tau)$, and leads to a point of tangency between $F(\tau)$ and $\theta(\tau) - \Delta I$ at a higher critical permit price τ^*. The higher CO_2 price is required to ensure τ won't in the future fall back below the level at which the dirty plant is optimal.

[41] In this and all subsequent figures, parameter values are as given in Table 2 unless otherwise noted.

[42] Recall equation 3 from Section 3.1: $Var[\tau(t)] = \tau_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1)$.

When $\tau < \tau^*$ and the firm's investment deadline has not been reached, then, increasing σ causes a general delay in any new plant investment.[43] In the context of this example, firms able to delay investment in the clean IGCC unit will do so until $\tau > \tau^*$, while firms that reach their investment deadline before τ exceeds τ^* will build the default coal plant. Thus, uncertain carbon emissions policy should cause both a general delay in capacity investment and a reduction in clean plant investment specifically, relative to the case where permit prices are certain and sufficiently high.

Figure 2: GBM τ^* as a Function of σ



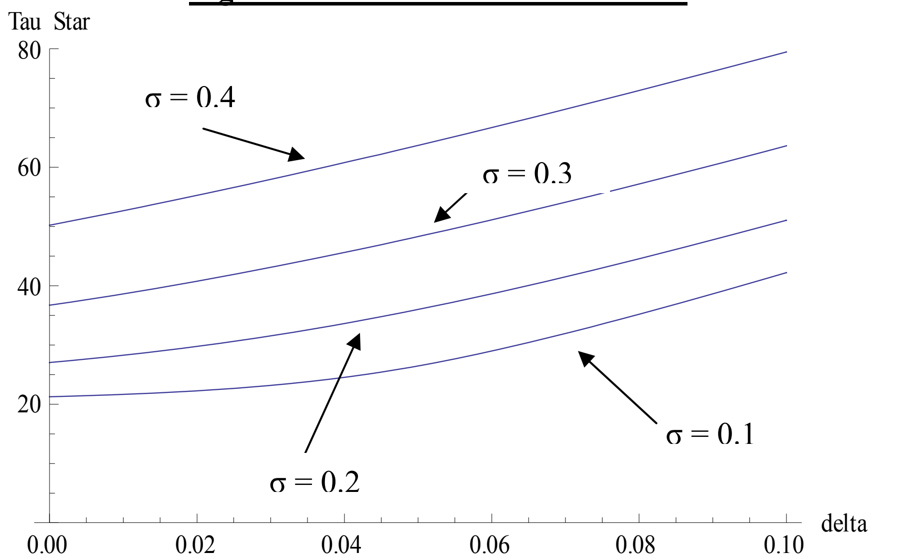
The dependence of τ^* explicitly on σ for different values of δ is presented in Figure 2. Again, τ^* is higher for higher values of σ. Also, recall that $\delta \equiv r - \alpha$, so that two thought experiments are possible: 1) assume r (the discount rate) is fixed and attribute changes in δ to changes in α (the drift rate of the CO_2 price's GBM process) and 2) assume α is fixed and attribute changes in δ to changes in r. When r is constant, increases in δ are due to reductions in α, implying that smaller values of α lead to a higher τ^*, an intuitive result given the equation for the expected value of the τ as provided in Eq. 2 in Section 3.1. as α is the drift parameter of τ's GBM process. The smaller the positive (or larger the negative) drift in τ, the smaller the expected excess future payoff from the clean plant, and the larger τ^* needs to be for clean investment to become optimal. Equivalently, a larger α implies a larger cost-savings for the clean plant, and clean investment becomes optimal at a lower τ^*.

Increasing the firm's discount rate, r, for a given value of α leads to a larger δ, and, again, a larger critical τ^* and a delay in clean-plant investment. Recall that the payoff from clean investment has been defined as the excess profit from clean (rather than dirty) plant investment, a function that is increasing in τ. The larger the firm's discount rate, the smaller the present value of this excess profit, and the larger is the τ^* that makes the clean investment optimal. These explanations for the effects of δ on τ^*, combined with the above-explained effects of σ on τ^*, account for the steeper increase in τ^* with larger values of δ.

[43] This is the standard result of option pricing models, and of the Dixit and Pindyck (1994) style investment-under-uncertainty models.

Figure 3 presents the dependence of τ^* on δ for different values of σ. The rate of increase in τ^* rises slightly as δ gets larger, and this rate of increase declines with σ. Recall that $\delta \equiv r - \alpha$. It could be assumed, following Dixit and Pindyck (1994) that r is constant, meaning that changes in δ are due to changes in α. As above, increases in δ occur when α falls, leading to an increase in τ^*. When $\delta > r$, $\alpha < 0$, indicating a negative drift rate in the CO_2 price, and a greater critical τ^* for clean-plant investment with a given σ. As σ increases, the effect of α on the slope of $\tau^*(\delta)$ in Figure 3 declines, as the variance in the CO_2 price overtakes the drift rate in importance in the determination of τ^*.

Figure 3: GBM τ^* as a Function of δ



5.2 Model 2: Jump-Diffusion

I now consider the dependence of the second model's results on the parameter assumptions related to τ's stochastic process. Here, τ evolves according to a jump-diffusion $d\tau = \alpha\tau dt + \sigma\tau dz + udq$. Recall that λ is the mean arrival rate of the permit price jumps, meaning that as λ increases, the amount of time between jumps in the permit prices $\left(\frac{1}{\lambda}\right)$[44] falls, so that for any given value of u, a larger λ leads to more frequent jumps in the permit price. This implies lesser downside risk for a given positive u, and greater downside risk for a given negative u. A similar interpretation can be applied to values of u for a given value of λ. When CO_2 price jumps are expected to be positive, a larger u for a given λ implies lower downside risk and therefore a lower τ^*. When u is negative, larger jumps are associated with increased downside risk and a higher τ^*.

Figures 4 and 5 depict how the critical τ^* depends on λ for negative and positive values of u, respectively. It is obvious from Figure 4 that a higher λ with a negative u implies a higher

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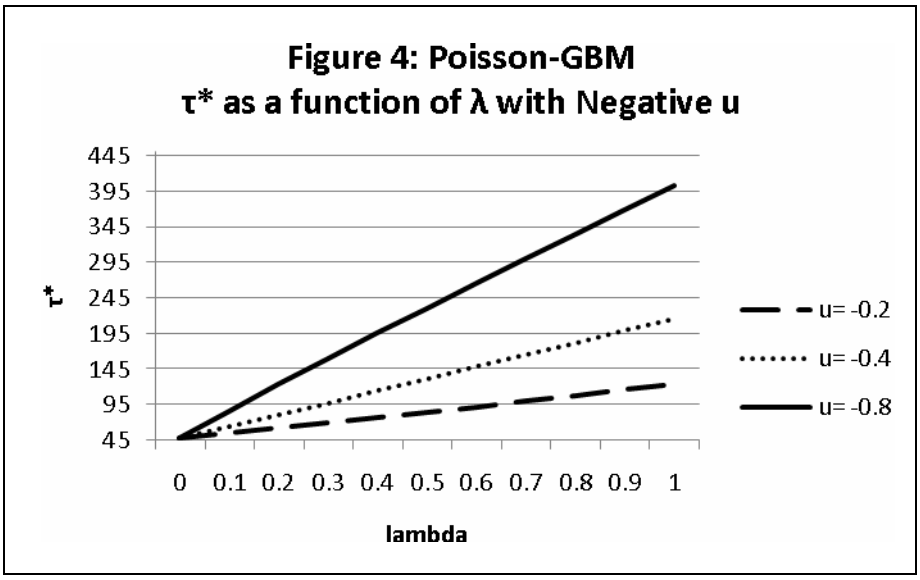
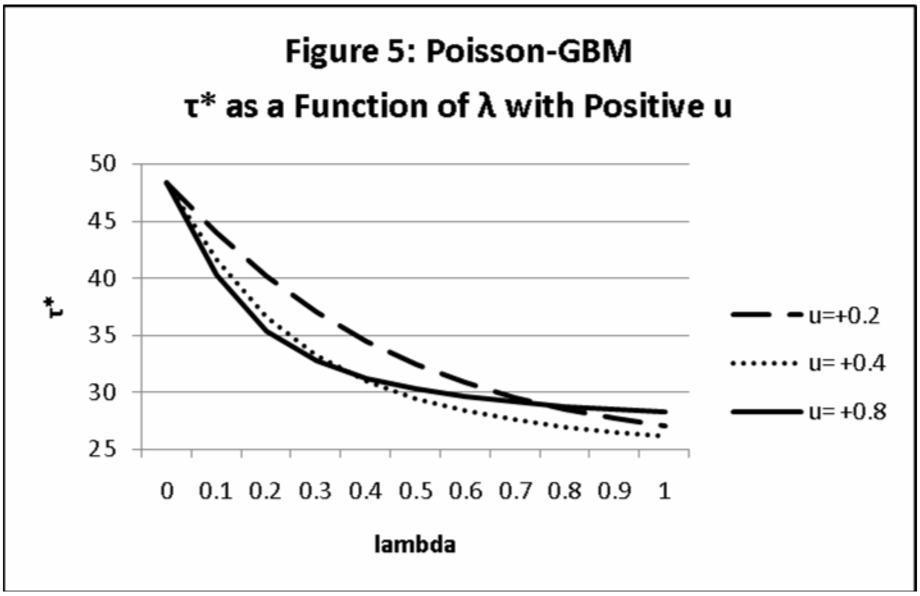
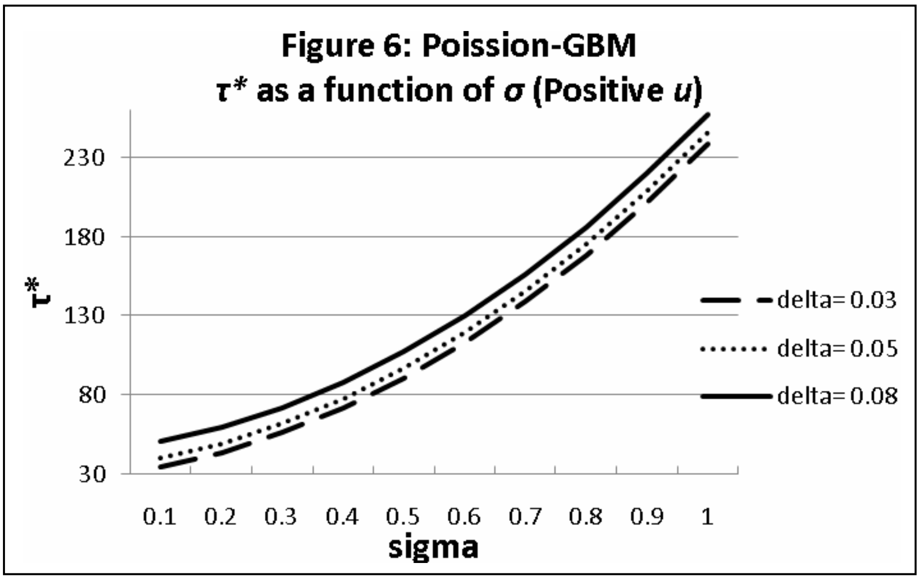
[44] This is a property of the Poisson process. The expected time until a jump is $\varepsilon[T] = \int_0^\infty \lambda T e^{-\lambda T} dt = \frac{1}{\lambda}$. See Dixit and Pindyck (1994) or Kannan (1979).

critical τ^*, and from Figure 5 that a higher λ with a positive u implies a lower critical τ^*. What is less obvious is that λ is affecting $\theta(\tau)$, the value of investing in a clean plant rather than a dirty one, in two ways. Remember from equation 22 in Section 4.2 that $\frac{1}{dt}\frac{E(d\tau)}{\tau} = \alpha + \lambda u$, meaning that an increase in λ leads to a larger expected percentage change in τ in each period (positive or negative, depending on u). This tends to increase the opportunity cost of investing in a clean plant now in the case of a negative u, and decrease the opportunity cost with a positive u. This in turn increases or decreases $F(\tau)$, respectively, and τ^* follows. Also, recall from Eq. 28 in Section 4.2 that $Var[d\tau] = \sigma^2\tau^2 dt + \lambda u^2\tau^2 dt$, which implies that increasing λ leads to an increase in the instantaneous variance of the percentage changes in τ, regardless of the sign of u. When u is negative, both of these influences work on τ in the same direction, and when u is positive, the work opposite each other. This explains the reduction in the decline in τ^* as λ increases in Figure 5, as a higher λ leads to a smaller decline in $F(\tau)$, and thus in τ^*.





The solid curve for u= +0.8 in Figure 5 illuminates a further interesting point. When both λ and the jump size are sufficiently large, the expected value of both investments (clean and dirty) declines, while the value of the option to invest in the clean plant, $F(\tau)$, increases. The result is a higher critical τ^* than in the case of smaller positive jumps, which explains the increasing convexity of the τ^* functions in Figure 5 for larger values of u.



Finally, Figure 6 depicts the dependence of τ^* on σ for different values of δ, for positive u values. The relationship is similar to the one found in the GBM model and for the same reasons, as CO_2 prices evolve here between jumps according to the same GBM as above. Thus, again, increasing σ for any value of δ again tends to increase the critical τ^* because increasing the variance of future permit prices increases the downside risk of the clean plant investment, which increases the value of the investment option relative to the payoff received from immediate investment. Intuitively, the greater the instantaneous variance of $d\tau$, the larger τ^* must be before the firm is convinced that future permit prices will not fall low enough to have made the dirty choice optimal. The graph showing τ^* as a function of σ for negative values of u is relegated to Appendix 6.

Both of these models show that increasing uncertainty over future emissions prices causes electricity generating firms to delay their plant-type investment decisions, and even to avoid clean investment when the critical τ^* is not reached before the firm's investment deadline arrives. The inclusion of the possibility of jumps in permit prices as the government learns more about the effects of CO_2 emissions was broken into two cases here: one in which price jumps were expected to be positive (associated with increasingly strict future emissions policy), and one where they were expected to be negative (associated with increasingly lenient future emissions policy). When the emissions cap is expected to tighten in the future, clean investment happens sooner relative to when prices are assumed to evolve according to a jumpless GBM process, but later than in the case of emissions price certainty (i.e. under a carbon tax scheme). If firms expect the emissions cap to loosen (because of security of supply concerns or government focus on growth or fuel poverty issues), then clean investment requires the highest critical τ^* and sees the longest delay, implying the greatest probability of investment in the dirty alternative when firms face investment deadlines. An interesting extension to this model would account for the case where the firm is not certain which direction permit prices might jump.

The fact that τ^* is lowest when emissions prices are certain indicates that clean investment is induced soonest in the case of an emissions tax. As previously noted, however, the government faces a similar problem regarding environmental uncertainty and policy timing, as the government learns more about the costs and benefits of environmental regulation over time. In such a situation, it may be optimal for the government to delay the establishment of environmental policy. (See, e.g., Pindyck (2000).) Note also that a welfare analysis of the various policies alluded to above has not been performed, but if capacity investment (and particularly clean capacity investment) was found to be desirable, the models presented above indicate that a credible certain carbon price in the form of an emissions tax would alleviate the investment delay problems caused by uncertain allowance prices. If, however, cap-and-trade regimes remain the most popular form of environmental regulation, the models here suggest that clean investment delay resulting from uncertain emissions prices could be reduced in one of two ways: 1) by setting a more stringent original cap while retaining the ability to relax emissions standards in the future, or 2) by establishing a less-stringent initial cap, but credibly relinquishing the right to its future alteration.

6 Conclusions

This paper presents two models that investigate the plant-type construction decision of an electricity generating firm under two alternative stochastic processes for CO_2 price uncertainty. These stochastic processes were chosen to represent varying degrees of regulatory policy commitment and were compared to the case in which permit prices are set and known with certainty (i.e. the case of a certain emissions tax). In the first model, the regulator is assumed to have established a fixed emissions cap under which permits are traded freely between firms, while in the second, the regulator maintains the right to alter the emissions cap in the future. Because of the irreversibility of a power plant investment and uncertainty over future emissions prices, both of the models are based on real options theory from the literature surrounding Pindyck (1991a,b), McDonald and Siegel (1986), and Dixit and Pindyck (1994) and both were solved using the method of dynamic programming.

The motivations behind this paper were related to the importance of electricity availability for sustained economic growth and to the uncertainty over potentially catastrophic climate events exacerbated by generation via dirty fossil fuels. Each successive government, then, is faced with the task of balancing the opposing objectives of economic growth, security of supply and environmental regulation, leaving future emissions prices and policy uncertain from the perspective of the firm. It is important to understand the impact of this uncertainty on a firm's plant-type investment decisions for the purpose of energy and policy planning. This decision, and the usefulness of real options analysis in modelling it, was therefore the focus of this paper.

Because generation via coal combustion is the most profitable in the absence of environmental regulations, and given the success and popularity of cap-and-trade emissions regulation (e.g. the CAAA 1990's SO_2 scheme in the US and the EU ETS), it was assumed that 1) a firm's default investment decision is a "dirty" coal combustion plant, and 2) firms are operating under a cap-and-trade programme for CO_2 emissions. In this setting, I sought the critical permit price, τ*, at which it becomes optimal for a firm to switch its investment choice from "dirty" to "clean".

The models presented in Section 3 made it clear that the possibility of reduced future permit prices creates an opportunity cost of immediate investment, thereby creating an option value for the firm trying to choose between its clean and dirty alternatives. In the GBM model, uncertainty over future permit prices is due entirely to the price volatility encountered through the free trading of the emissions permits. In the jump-diffusion model, permit prices are also able to jump at random intervals as the regulating body alters future CO_2 caps.
The results, presented in Section 4, show that uncertainty over future emissions policy causes a general delay in all investment, and a reduction in clean plant investment. As seen in the example provided, this delay is most pronounced in the jump-diffusion model when firms expect the emissions cap to become looser at some point in the future. Here, the clean-investment-inducing τ* was more than twice that of the stationary deterministic (tax) reference case, and 14% larger than in the GBM case. When permit prices were modelled as following a GBM, the effect was still quite large, with τ* approximately twice as large as in the stationary deterministic case. When emissions caps were expected to tighten in the jump-diffusion model, the critical τ* was lower than in the GBM case, but still higher than in the reference certainty case.

The results imply that uncertain environmental policy leads to a reduction in investment in clean generating capacity and a delay in capacity investment in general, particularly when it is possible for emissions regulation to become less stringent in the future. Note that the government may prefer to retain flexibility in the prescribed environmental policy in order to assimilate information about the future costs and benefits of environmental regulation, and the establishment of any particular emissions policy is not being prescribed without a complete welfare analysis. However, the results indicate that if timely capacity investment and emissions regulation were found to be desirable, a known and certain emissions tax may be the most appropriate form for environmental policy to take. The models show that in the case of a cap-and-trade scheme, the effects of uncertainty are most pronounced when the government retains the right to loosen emissions caps in the future, implying that the establishment of a sufficiently stringent policy to which the government will remain credibly committed would be the most effective means of increasing the quantity of clean generating capacity in the US, should that be found to be desirable.

Several useful and interesting extensions to this work are possible, including the incorporation of construction lags, construction subsidies and project financing schemes, the analysis of additional unit-specific decisions, and the effects of a standard jump-diffusion process where u could be either positive or negative. The incorporation of uncertainty over prices and quantities of future inputs and outputs would also make an interesting study, as would the parallel analysis of the policy timing decision of the government.

Appendix 1- Derivation of the continuous-time Bellman

In discrete time the firm's objective is make the investment decision such that

$$F_t(\tau_t) = \max\left\{\frac{1}{1+r} E_t[F_{t+1}(\tau_{t+1})]\right\}. \qquad \textbf{Eq. A1.1}$$

We are working in continuous time, however, meaning that the firm will be concerned with the rate of change of the current payoff and the continuation value per unit time. Thus, if Δt is the length of the time period, r is the discount rate per unit time and τ' represents the value of τ in the next period, the firm's objective is

$$F(\tau) = \max\left(\frac{1}{1+r\Delta t}\right)E[F(\tau')].$$

Multiplying both sides by $1 + r\Delta t$ and rearranging,

$$r\Delta t F(\tau) = \max E[\Delta F(\tau)].$$

Dividing by Δt and letting $\Delta t \to 0$ so that time is continuous, the firm's objective of

$$rF(\tau) = \max \frac{1}{dt} E[dF(\tau)], \qquad \textbf{Eq. A1.2}$$

is revealed, leaving Bellman Eq. 11 in Section 3.1 for this problem as follows:

$$rF(\tau) = \frac{1}{dt} E[dF(\tau)]. \qquad \textbf{Eq. A1.3}$$

Equation A1.3 simply states that the normal rate of return on the option to invest in a clean plant must equal the expected change in the value of the option. In other words, the total expected return on the option is just equal to the option's expected capital gain per unit of time.

Appendix 2- Model 1 Derivation of $F(\tau)$ differential equation from Bellman Equation 11

Recall the Bellman Eq. 11 for the value of the investment option given in Section 3.1 and in Eq. A1.3 on the previous page:

$$rF(\tau) = \frac{1}{dt} E[dF(\tau)]. \qquad \textbf{Eq. A2.1}$$

From equation A2.1 the differential equation for $F(\tau)$, the value of the clean-plant investment option, is gleaned. In order to proceed, expand the right hand side of Eq. A2.1 as follows:

$$dF(\tau) = \frac{\partial F(\tau)}{\partial t} dt + \frac{\partial F(\tau)}{\partial \tau} d\tau .$$

Because $F(\tau)$ is not an explicit function of time in this case, $\frac{\partial F(\tau)}{\partial t} = 0$. Using Ito's Lemma and eliminating all elements containing dt to a power greater than one,

$$dF(\tau) = \alpha\tau F'(\tau)dt + \sigma\tau F'(\tau)dz + \frac{1}{2}\sigma^2\tau^2 F''(\tau)dt. \qquad \textbf{Eq. A2.2}$$

Taking the expectation, and recalling that $E[dz] = 0$,

$$E[dF(\tau)] = \frac{1}{2}\sigma^2\tau^2 F''(\tau)dt + \alpha\tau F'(\tau)dt. \qquad \textbf{Eq. A2.3}$$

Plugging equation (A2.3) into (A2.1), the Bellman yields

$$rF(\tau) = \frac{1}{dt}\left[\frac{1}{2}\sigma^2\tau^2 F''(\tau)dt + \alpha\tau F'(\tau)dt\right]$$

which can be rearranged and simplified to reveal the differential equation for $F(\tau)$, presented as Eq. 12 in Section 3.1:

$$0 = \frac{1}{2}\sigma^2\tau^2 F''(\tau) + (r-\delta)\tau F'(\tau) - rF(\tau), \qquad \textbf{Eq. A2.4}$$

Where, recalling that $\delta \equiv r - \alpha$, α has been replaced with r - δ in order to eliminate one parameter from the calculations.

Appendix 3- Derivation and solution of the characteristic equation for the solution of differential Eq. 12 in Model 1

The characteristic equation that yields β_1 and β_2 is just the equation that results from plugging the solution form $F(\tau) = A\tau^{\beta}$ into the differential equation, repeated here for convenience:

$$0 = \frac{1}{2}\sigma^2\tau^2 F''(\tau) + (r-\delta)\tau F'(\tau) - rF(\tau). \qquad \textbf{Eq. A3.1}$$

When $F(\tau) = A\tau^{\beta}$, $F'(\tau) = \beta A\tau^{\beta-1}$ and $F''(\tau) = \beta(\beta-1)A\tau^{\beta-2}$. Substituting into the homogeneous portion of Eq. A3.1,

$$\frac{1}{2}\beta(\beta-1)A\tau^{\beta-2}\sigma^2\tau^2 + (r-\delta)\tau\beta A\tau^{\beta-1} - rA\tau^{\beta} = 0$$

which, because $F(\tau) = A\tau^{\beta}$, is just

$$\frac{1}{2}\beta(\beta-1)\sigma^2 F(\tau) + (r-\delta)F(\tau) - rF(\tau) = 0$$

or

$$\left[\left(\frac{1}{2}\beta(\beta-1)\sigma^2\right) + \left(\left(r-\delta\right)\beta\right) - r\right]F(\tau) = 0. \qquad \textbf{Eq. A3.2}$$

Thus, in order for $F(\tau) = A\tau^{\beta}$ to be a solution to the homogeneous portion of Eq. 12 in Section 3.1 (and Eq. A3.1 above), the following must be true

$$\frac{1}{2}\beta(\beta-1)\sigma^2 + \left(r-\delta\right)\beta - r = 0.$$

Collecting terms in β, the characteristic equation is revealed:

$$\frac{1}{2}\sigma^2\beta^2 + (r-\delta-\frac{1}{2}\sigma^2)\beta - r = 0. \qquad \textbf{Eq. A3.3}$$

The two roots of this quadratic equation are

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \frac{\sqrt{(r-\delta)^2 + (\delta - r + \frac{1}{4}\sigma^2 + 2r)\sigma^2}}{\sigma^2} > 0 \qquad \textbf{Eq. A3.4}$$

and

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \frac{\sqrt{(r-\delta)^2 + (\delta - r + \frac{1}{4}\sigma^2 + 2r)\sigma^2}}{\sigma^2} < 0 \qquad \textbf{Eq. A3.5}$$

Appendix 4- Model 2 Case 2 derivation of differential equation for $F(\tau)$ when τ follows a jump-diffusion process

Recall the Bellman Eq. 31 for the investment problem from Section 3.2

$$rF(\tau) = \frac{1}{dt} E[dF(\tau)]. \qquad \textbf{Eq. A4.1}$$

The jump-diffusion process τ follows in this model requires us to adopt a different version of Ito's lemma to expand the right hand side of Eq. A4.1. Here, as in Model 1,

$$dF(\tau) = \frac{\partial F(\tau)}{\partial t} dt + \frac{\partial F(\tau)}{\partial \tau} d\tau,$$

but from Eq. 20, $d\tau = \alpha\tau dt + \sigma\tau dz + \tau dq$, and F(τ) is not an explicit function of time, meaning $\frac{\partial F(\tau)}{\partial t} = 0$. $\frac{\partial F(\tau)}{\partial \tau} d\tau$ can then be broken into two parts: one due to the geometric Brownian motion portion of Eq. 20, which can be gleaned from the standard version of Ito's Lemma, and the second due to the jump process in Eq. 20. Thus,

$$E[dF(\tau)] = \alpha\tau F'(\tau)dt + \frac{1}{2}\sigma^2\tau^2 F''(\tau)dt + \lambda[F((1+u)\tau) - F(\tau)]dt$$

$$= \alpha\tau F'(\tau)dt + \frac{1}{2}\sigma^2\tau^2 F''(\tau)dt + \lambda F[(1+u)\tau]dt - \lambda F(\tau)dt \qquad \textbf{Eq. A4.2}$$

Substituting equation (A4.2) into the right hand side of equation (A4.1),

$$rF(\tau)dt = \alpha\tau F'(\tau)dt + \frac{1}{2}\sigma^2\tau^2 F''(\tau)dt + \lambda F[(1+u)\tau]dt - \lambda F(\tau)dt.$$

Dividing by *dt* and rearranging, the differential equation for *F*(*τ*) is revealed:

$$0 = \frac{1}{2}\sigma^2\tau^2 F''(\tau) + \alpha\tau F'(\tau) + \lambda F[(1+u)\tau] - (\lambda + r)F(\tau). \qquad \textbf{Eq. A4.3}$$

Appendix 5- Characteristic equation for Model 2 Case 2

Recall the differential equation given by Eq. 32 in Section 2.2:

$$0 = \frac{1}{2}\sigma^2\tau^2 F''(\tau) + \alpha\tau F'(\tau) + \lambda F[(1+u)\tau] - (\lambda + r)F(\tau), \qquad \textbf{Eq. A5.1}$$

which will have a solution of the form

$$F(\tau) = A_1\tau^{\beta_1}. \qquad \textbf{Eq. A5.2}$$

Here, as in the Appendix 3,

$$F(\tau) = A_1 F^{\beta_1},\ F'(\tau) = \beta_1 A_1 \tau^{\beta_1 - 1} \text{ and } F''(\tau) = \beta_1(\beta_1 - 1)A_1\tau^{\beta_1 - 2}. \qquad \textbf{Eq. A5.3}$$

Inserting equations A5.3 into the differential equation A5.1,

$$\frac{1}{2}\sigma^2\tau^2\left[\beta_1(\beta_1 - 1)A_1\tau^{\beta_1 - 2}\right] + \alpha\tau\left[\beta_1 A_1\tau^{\beta_1 - 1}\right] + \lambda A_1[(1+u)\tau]^{\beta_1} - (\lambda + r)A_1\tau^{\beta_1} = 0,$$

which, because $F(\tau) = A_1\tau^{\beta_1}$, is just

$$\frac{1}{2}\sigma^2\beta_1(\beta_1 - 1)F(\tau) + \alpha\beta_1 F(\tau) + \lambda(1+u)^{\beta_1}F(\tau) - (\lambda + r)F(\tau) = 0$$

or

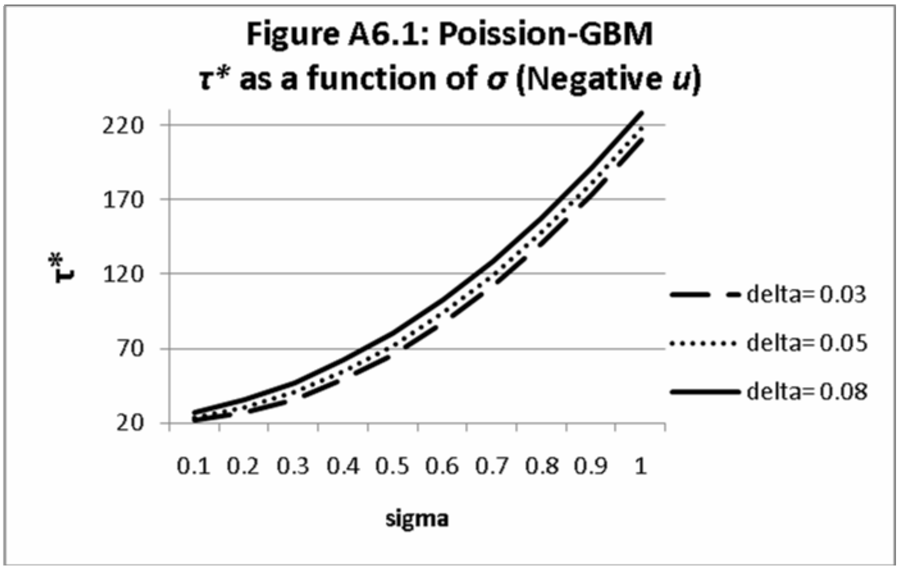
$$\left[\frac{1}{2}\sigma^2\beta_1(\beta_1 - 1) + \alpha\beta_1 + \lambda(1+u)^{\beta_1} - (\lambda + r)\right]F(\tau) = 0.$$

Thus, in order for $F(\tau) = A_1\tau^{\beta_1}$ to be a solution to Eq. 32, β_1 must solve

$$\frac{1}{2}\sigma^2\beta_1(\beta_1 - 1) + (r - \delta)\beta_1 + \lambda(1+u)^{\beta_1} - (\lambda + r) = 0, \qquad \textbf{Eq. A5.4}$$

where, again, α has been replaced with $r - \delta$. The solution for β_1 from Eq. A5.4 can not be found analytically and must therefore be solved numerically.

Appendix 6: τ as a function of σ with negative u*



Bibliography

Aitchison, J. and J. A. C. Brown (1957), *The Lognormal Distribution,* (Cambridge: Cambridge University Press).

Arrow, K. and A. Fisher (1974), "Environmental Preservation, Uncertainty and Irreversibility", *Quarterly Journal of Economics,* 88(2): 312-319.

Beamon, J. A. and T. J. Leckey (1999), "Trends in Power Plant Operating Costs", From *Issues in Midterm Analysis and Forecasting* Report No. EIA/DOE-0607(99), Energy Information Administration. http://www.eia.doe.gov/oiaf/issues/power_plant.html.

Black, F. and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, 81(3): 637-654.

Blyth, W., R. Bradley, D. Bunn, C. Clarke, T. Wilson, and M. Yang (2007), "Investment Risks Under Uncertain Climate Change Policy", *Energy Policy*, 35: 5766-5773.

Bulan, L. (2005), "Real Options, Irreversible Investment and Firm Uncertainty: New Evidence from US Firms", *Review of Financial Economics*, 14(3-4): 255-279.

Cicchetti, C. J. and A. M. Freeman III (1971), "Option Demand and Consumer Surplus: Further Comment", *Quarterly Journal of Economics,* 85(3): 528-539.

Clarkson, R. and K. Deyes (2002), "Estimating the Social Cost of Carbon Emission", GES Working Paper 140. London: HM Treasury.

Cox, D. R. and H. D. Miller (1965), *The Theory of Stochastic Processes*, (London: Methuen & Co. Ltd.)

Cox, J. and M. Rubinstein (1985), *Option Markets,* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc.)

Dasgupta, P. and G. Heal (1974), "The Optimal Depletion of Exhaustible Resources", *Review of Economic Studies,* Special Issue, 41(128): 3-28.

Dasgupta, P. and G.Heal (1979), *Economic Theory and Exhaustible Resources,* (Welwyn: James Nisbet and Co. Ltd and Cambridge University Press.)

Devarajan, S. and A. C. Fisher (1982), "Exploration and Scarcity", *Journal of Political Economy*, 90(6): 1279-90.

Dixit, A. K. and R. S. Pindyck (1994), *Investment Under Uncertainty,* (Chichester, West Sussex: Princeton University Press).

Dixit, A. (1990), *Optimization in Economic Theory*, 2nd Edition. (Oxford: Oxford University Press).

Dixit, A. (1993), *The Art of Smooth Pasting*, Vol. 55 in *Fundamentals of Pure and Applied Economics*, eds. Jacques Lesourne and Hugo Sonnenschein Chur, The Netherlands: Harwood Academic Publishers.

EnergyInformation Administration. Electric Power Industry. www.eia.doe.gov/cneaf/electricity/page/prim2/chapter1.html. Chapters 1-6.

EIA (2005), *Electric Power Annual*. Available online at http://www.eia.doe.gov/cneaf/electricity/epa/epa.pdf.

EIA (2004b), *Assumptions for the Annual Energy Outlook 2004 with Projections to 2025*. Report no. DOE/EIA-0554(2004). Available online at http://www.eia.doe.gov/oiaf/aeo/assumption/pdf/0554(2004).pdf.

EIA (2004a), *Annual Energy Outlook 2004 with Projections to 2025.* Report no. DOE/EIA-0383(2004). Available online at http://www.eia.doe.gov/oiaf/aeo/index.html.

EIA, *Electric Power Industry Restructuring Fact Sheet*. Last modified 3 September 2002. Available online at http://www.eia.doe.gov/cneaf/electricity/page/fact_sheets/restructuring.html.

EIA (1995), *Electricity Generation and Environmental Externalities: Case Studies*. Report no. DOE-EIA 0598. Available online at http://www.eia.doe.gov/cneaf/electricity/external/external.pdf.

EIA (2007), "Electric Power Annual 2006", Report No: DOE/EIA 0348(2006). Available at: http://www.eia.doe.gov/cneaf/electricity/epa/epa.pdf.

EIA and EPA (2000), *Carbon Dioxide Emissions from the Generation of Electric Power in the United States*. Available online at http://tonto.eia.doe.gov/FTPROOT/environment/co2emiss00.pdf.

EPA (1999), *Emissions Factors, Global Warming Potentials, Unit Conversions, Emissions, and Related Facts*, complied by ICF Consulting. Available online at http://www.epa.gov/appdstar/pdf/brochure.pdf.

EPA, *Clean Air Markets- Programs and Regulations – Acid Rain Program: An Overview*. Las updated 10 May 2004. Available online at http://www.epa.gov/airmarkets/arp/overview.html

EPA, *Clean Energy- How Does Energy Use Affect the Environment?* Last updated 5 February 2004. Available online at http://www.epa.gov/cleanenergy/impacts.htm

EPA, *Clean Air Markets- Allowance Trading-Allowance Prices (1995-2002)*. Last updated 11 February 2004. Available online at http://www.epa.gov/airmarkets/trading/so2market/alprices.html

Eyre, N., T. Downing, R. Hoekstra, K. Rennings, and R. Tol (1997), *Global Warming Damages*, Final Report to the ExternE Global Warming Sub-Task, DGXII, European Commission, Brussels.

Federal Reserve (26 March 2004) *Selected Interest Rates: Historical Data: Treasury Constant Maturities: 20-year: Annual*. Available online at http://www.federalreserve.gov/releases/h15/data/a/tcm20y.txt.

Fisher, A. C. (2000) "Investment Under Uncertainty and Option Value in Environmental Economics", *Resource and Energy Economics,* 22(3): 197-204.

Hansen, U. (1998), "Technological Options for Power Generation", *Energy Journal,* 19(2): 63-87.

Hanemann, M. W. (1989), "Information and the Concept of Option Value", *Journal of Environmental Economics and Management,* 16: 23-37.

Hassett, K. A. and G. E. Metcalf (1999), "Investment With Uncertain Tax Policy: Does Random Tax Policy Discourage Investment?", *The Economic Journal,* 109 (July): 372-393.

Helm, D. (2002), "Energy Policy: Security of Supply, Sustainability and Competition", *Energy Policy,* 30(3): 173-184.

Helm, D. and C. Hepburn and R. Mash (2003), "Credible Carbon Policy", *Oxford Review of Economic Policy*, 19(3): 438-450.

Henry, C. (1974), "Option Values in the Economics of Irreplaceable Assets", *Review of Economic Studies*, Symposium, (0): 89-104.

Herbelot, O. (1994), "Option Valuation of Flexible Investments: The Case of a Scrubber for Coal-Fired Power Plant", Massachusetts Institute for Technology Working Paper No. WP-94001.

Herbelot, O. (1992), "Option Valuation of Flexible Investments: The Case of Environmental Investments in the Electric Power Industry", Unpublished PhD dissertation, Massachusetts Institute of Technology, May.

Hugonnier, J. and E. Morellec (2007), "Real Options and Risk Aversion", Swiss Finance Institute Research Paper. Available at SSRN: http://ssrn.com/abstract=422600.

Insley, M. C. (2003), "On the Option to Invest in Pollution Control Under a Regime of Tradable Emissions Allowances", *Canadian Journal of Economics,* 36(4): 860-883.

Ishii, J. and J. Yan (2004), "Investment Under Regulatory Uncertainty: US Electricity Generation Investment Since 1996", Center for the Study of Energy Markets, University of California Energy Institute, Working Paper CSEM WP 127.

Jorgenson, D. W. (1963), "Capital Theory and Investment Behavior", *American Economic Review,* 53 (May): 247-259.

Jorgenson, D. W. (1971), "Econometric Studies of Investment Behavior: A Survey", *Journal of Economic Literature,* 9: 111-147.

Joskow, P. L., R. Schmalensee, and E. M. Bailey (1998), "The Market for Sulfur Dioxide Emissions", *The American Economic Review*, 88: 669-685.

Kannan, D. (1979), *An Introduction to Stochastic Processes,* (New York: North Holland).

Kiriyama, E. and A. Suzkuki (2004), "Use of Real Options in Nuclear Power Plant Valuation in the Presence of Uncertainty with CO_2 Emissions Credit", *Journal of Nuclear Science and Technology*, 41(7): 756-764.

Kolstad, C. D. (1996a), "Learning and Stock Effects in Environmental Regulation: The Case of Greenhouse Gas Emissions", *Journal of Environmental Economics and Management,* 31: 1-18.

Kolstad, C. D. (1996b), "Fundamental Irreversibilities in Stock Externalities", *Journal of Public Economics,* 60: 221-233.

Laurikka, H. and T. Kolgonen (2006), "Emissions Trading and Investment Decisions in the Power Sector: A Case Study of Finland", *Energy Policy*, 34: 1063-1074.

Laurikka, H. (2006), "Option Value of Gasification Technology Within an Emissions Trading Scheme", *Energy Policy*, 34(18): 3916-3928.

Leahy, J. V. and T. M. Whited (1995), "The Effect of Uncertainty on Investment: Some Stylized Facts", NBER Working Paper No. 4986.

Malliaris, A. G. and W. A. Brock (1982), "Stochastic Methods in Economics and Finance", Advanced Textbooks in Economics Series, ed. C.J. Bliss and M.D. Intriligator, No. 17. Oxford: North-Holland Publishing Company.

McDonald, R. and D. Siegel (1986), "The Value of Waiting to Invest", *The Quarterly Journal of Economics,* November: 707-727.

Endelsohn, R. and J. Neumann (eds.), (1999), *The Impact of Climate Change on the US Economy,* (Cambridge: Cambridge University Press).

Merton, R. C. (1971), "Optimal Consumption and Portfolio Rules in a Continuous-Time Model", *Journal of Economic Theory*, 3: 373-413.

Merton, R. C. (1976), "Option Pricing When Underlying Stock Returns are Discontinuous", *Journal of Financial Economics*, 3: 125-144.

Montero, J.P. (1999), "Voluntary Compliance with Market-Based Environmental Policy: Evidence from the US Acid Rain Program", *Journal of Political Economy*, 107: 998-1033.

Pearce, D. (2003), "The Social Costs of Carbon and its Policy Implications", *Oxford Review of Economic Policy*, 19: 362-384.

Pearce, D. W., W.R. Cline, A. Achanta, S. Fankhauser, R. Pachauri, R. Tol and P. Vellinga, (1996), "The Social costs of Climate Change: Greenhouse Damage and the Benefits of Control", In Intergovernmental Panel on Climate Change, *Climate Change 1995: Economic and Social Dimensions of Climate Change*, (Cambridge: Cambridge University Press), pp. 183-224.

Perman, R., Y. Ma, J. McGilvray and M. Common (1999), *Natural Resource and Environmental Economics*, Second Edition, (Essex: Pearson Education Limited.)

Pindyck, R. S. (1978), "The Optimal Exploration and Production of Nonrenewable Resources", *Journal of Political Economy*, 86(5): 841-861.

Pindyck, R. S. (1991a), "Irreversibility and the Explanation of Investment Behavior," in *Stochastic Models and Option Values: Applications to Resources, Environment and Investment Problems,* Contributions to Economic Analysis Series, ed. D. Lund and B. Øksendal (Amsterdam: Elsevier Science Publishers B.V.)

Pindyck, R. S. (1991b), "Irreversibility, Uncertainty, and Investment", *Journal of Economic Literature*, 29: 1110-1148.

Pindyck, R. S. (1993), "Investments of Uncertain Cost", *Journal of Financial Economics*, 34(1): 53-76.

Pindyck, R. S. (2000), "Irreversibilities and the Timing of Environmental Policy", *Resource and Energy Economics*, 22: 233-259.

Rabl, A. and J. V. Spadro (2000), "Public Health Impacts of Air Pollution and Implications for the Energy System", *Annual Review of Energy and the Environment,* 25: 601-627.

Romer, D. (2001), *Advanced Macroeconomics*, 2nd Edition, (London: MacGraw Hill Higher Education.)

Roughgarden, T. and S. Schneider (1999), "Climate Change Policy: Quantifying Uncertainties for Damages and Optimal Carbon Taxes", *Energy Policy*, 27: 425-429.

Schmalensee, R., P. J. Loskow, A. D. Ellerman, J. P. Montero, and E. M. Bailey (1998), "An Interim Evaluation of Sulfur Dioxide Emissions Trading", *Journal of Economic Perspectives,* 12, Summer: 53-68.

Söderholm, P. (1998), "The Modelling of Fuel Use in the Power Sector: A Survey of Econometric Analyses", *The Journal of Energy Literature*, IV(2): 3-27.

Stavins, R. N. (2003), "Market-Based Environmental Policies: What can we Learn from the US Experience (and Related Research)?", Resources for the Future, Discussion Paper 03-43.

Stiglitz, J.E. (1974), "Growth with Exhaustible Natural Resources", *Review of Economic Studies*, Special Issue, 41(128): 123-137.

Tol, R. and T. Downing (2000), *The Marginal Costs of Climate Changing Emissions*, Institute for Environmental Studies: Free University of Amsterdam.

Tuthill, L. (2007), "The Fuel Choice and Technological Change Effects of the Tradable Sulfur Permit Scheme on the US Electricity Generating Industry," in C.C. Aktan and S. Balta (eds.) *Perspectives on Economics, Vol. 1*, Selected Proceedings of the Third International Conference on Business, Management and Economics.

Tuthill, L. (2008), "Interfuel Substitution in the US Electricity Generating Industry Under the Tradable Sulphur Allowance Scheme: 1990-2004", Oxford Institute for Energy Studies, Working Paper EV 40.

VanVactor, S. A. (2003), "Flipping the Switch: The Transformation of Energy Markets", D.Phil. Thesis, Scott Polar Research Institute, University of Cambridge.

Vickson, R.G. (1975), "Appendix A. An Intuitive Outline of Stochastic Differential Equations and Stochastic Optimal Control" in *Stochastic Optimization Models in Finance*, ed. W.T. Ziemba and R.G. Vickson (London: Academic Press).

Wilmott, P., J. Dewynne and S. Howison (1993), *Option Pricing: Mathematical Models and Computation*, (Oxford: Oxford Financial Press.)

Xepapadeas, A. (1999), "Environmental Policy and Firm Behavior: Abatement Investment an Location Decisions Under Uncertainty and Irreversibility", NBER Technical Working Paper No. 243.

Zervos, M., B. Meister and T. Knudsen (1999), "On the Relationship of the Dynamic Programming Approach and the Contingent Claims Approach to Asset Valuation", *Finance and Statistics,* 3: 433-449.