Redefining the Convenience Yield in the North Sea Crude Oil Market

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CONTENTS

Page

FIGURES

Page

TABLES

Page

ABSTRACT

The objective of this paper is to adapt the classical spot/futures relationship stemming from the theory of storage to the case of the North Sea crude oil market. Brent (and other North Sea crude oil grades) is waterborne and these logistical issues mean that a true spot market for Brent does not exist. As a result, the dated Brent (BFO) price, one of the most widely quoted 'spot' crude oil marker prices in the world, is in reality a short-term forward contract price. A new arbitrage relationship is constructed to take account of this and a new formulation for the convenience yield is calculated. Comparison is made with the classical convenience yield in the light of historic price behaviour in both Brent (BFO) and West Texas Intermediate (WTI) markets.

1. INTRODUCTION

Despite falling production volumes and recent modifications to the way it is traded and prices reported, Brent crude oil produced in the North Sea remains the dominant marker grade for most of the crude oil produced and consumed outside of the Americas. This paper derives from a fundamental observation made by Paul Horsnell and Robert Mabro[1] which is that 80% of the physical, so called, *dated* Brent cargoes traded are still to be loaded at the time of the transaction. From its inception, the logistics of the Brent market meant that there was an average gap of roughly thirteen days between a dated Brent contract agreement being made and the first day of the three-day date range when the parcel of oil is to be loaded on to the vessel. Recent contract reforms now mean the gap has been extended to an average gap of 17 days. This is not the case, for example, in the market for West Texas Intermediate (WTI) which is the dominant crude oil marker grade for the Americas. Spot WTI is traded for immediate delivery. Given the importance of dated Brent crude as a marker, this observation has an important impact on the way we represent the term structure of spot and forward prices of Brent crude. Horsnell and Mabro did not develop their observation further but it has implications for the classical formulation of convenience yield and the theory of storage in relation to the North Sea oil market.

The peculiarities of the market for Brent and other North Sea crude oil grades lead to the following fundamental questions:

i. Why does the classical cost of carry equation not reflect the term structure of prices in the case of Brent and other North Sea oil grades?

ii. What alternative formulation for the convenience yield could better explain the term structure of prices?

iii. What alternative formulation for the convenience yield could better explain the relationship between dated Brent and spot WTI?

The primary contribution of this paper is to provide a new formulation for the convenience yield in the Brent crude oil market within the framework of the theory of storage. We do not attempt to find a model for the term structure of prices in order to fit

1 Horsnell and Mabro, 1993, page 114 (see Table 8.1).

historic data because a number of studies have shown that the convenience yield evolves through time as a stochastic process (c.f. Gibson and Schwartz, 1990). Instead, we derive a theoretical formulation for the convenience yield that provides a better explanation of the specific characteristics of the Brent crude oil market.

1.1. The Classical Theory of Storage

The relationship between spot and futures (forward) commodity prices is the *basis*. If the futures (or forward) price of a commodity is greater than spot price the market is said it to be in *contango*. In general, the greater the contango the greater the incentive for producers and consumers to increase their holding of stocks (inventories), however, if the market is in backwardation, when the spot price is greater than the futures price, producers and consumers will have an increasing incentive to minimise stock levels. There are two complementary theories that explain the basis:

i. The normal backwardation theory (Keynes 1923); and

ii. The theory of storage.

The normal backwardation theory explains the basis from the point of view of hedging positions in the futures markets, and will not be discussed further, whereas the classical theory of storage is based on an arbitrage approach which, as the name suggests, focuses on the cost of holding physical stocks of a commodity.

The theory of storage is a simple tool to explain the term structure of prices because in the presence of surplus stocks, the level of contango cannot exceed the cost of carry otherwise risk free 'cash and carry' arbitrage trading operations would reestablish the equilibrium as expressed in [1] where the futures price $F(t,T)$ of maturity T at date t must always be greater or at least equal to the spot price $S(t)$ because of the costs incurred while storing physical stocks of commodity, the *cost of carry*, is represented by the interest rate r and other storage costs c:

$$F(t,T) = S(t)\exp[(r+c)(T-t)] \qquad [\ 1]$$

In the case of surplus stocks a backwardation is impossible because ‘reverse cash and carry’ trading operations would restore the equilibrium. The cash and carry arbitrage presented in Table 1 shows how the spot/futures equilibrium relationship expressed by the theory of storage is constructed.

Table 1: Classical arbitrage portfolio without convenience yield

Operations	Debits and credits in the portfolio at time t	Debits and credits in the portfolio at time T
Buy the physical asset	- S(t)	
Borrow the amount to pay the physical asset and pay the interest to the bank and the commodity insurance at time T	S(t)	-S(t)*[1+ (r+c)(T-t)]
Sell a futures contract whose maturity is (T-t) at time t and deliver the physical asset at time T	Value(F(t,T))=0*	F(t,T)
Imbalance	0	0

* The value of a futures contract is always 0 at the beginning of each trading day

A risk free arbitrage strategy must always provide a non-negative gain to a trader beginning with an initial zero endowment. The sum of the cash flows at time *T* must therefore be equal to 0 to ensure that there is no arbitrage opportunity, so $F(t,T) = S(t)*[1+(r+c)(T-t)]$ and in continuous time we obtain the formula in [1].

The classical theory of storage cannot adequately explain the frequently observed term structure of prices which occurs when stocks are scarce and the market becomes deeply backwardated (i.e. the spot price is significantly greater than the futures price). For the equilibrium relationship in [1] to hold, the cost of carry must theoretically become negative. This is logically and practically impossible because none of the underlying components that make up the cost of carry (i.e. interest rates, insurance, shrinkage, transit and warehousing fees) can individually or collectively take on a negative value. Therefore, a modified version of the theory of storage is required.

When stocks are abundant the cost of carry relationship holds, whereas in the case of stock scarcity the market will tend towards backwardation as reverse cash and carry trades become increasingly impossible to execute. The basis will change according to prevailing market conditions and there is no upper bound limit on the magnitude of the backwardation. Thus, in practice, the real spot/forward relationship in a commodity market is not an equilibrium one as expressed in the theory of storage but the inequality [2] as follows:

$$F(t,T) \leq S(t)\exp[(r+c)(T-t)] \qquad [\ 2]$$

This inequality is only valid in the case of real assets, which include commodities such as crude oil and other industrial raw materials and agricultural products, but not financial assets such as equities and bonds, where the classical theory of storage always holds. To address the apparent conflict between the classical theory of storage and empirical observation of real asset spot and futures prices the concept of a *convenience yield* first appeared in the theoretical debate with Kaldor (1939). This is expressed in the modified spot/futures arbitrage formulation [3] as follows:

$$F(t,T) = S(t)\exp[(r+c-cy)(T-t)] \qquad [\ 3]$$

Here, the marginal convenience yield cy is expressed as a percentage of the spot price (i.e. not in monetary value per unit of commodity) accruing to the owner of a spot commodity and now outweighs the cost of carrying the stocks $r + c$. Thus, the classical theory of storage, modified by the addition of the cy term now appears to hold under all empirically observed market conditions of contango and backwardation.

Rearranging the equilibrium relationship in [3] allows the convenience yield to be calculated as a percentage value:

$$cy = \frac{-1}{T-t}\ln\left(\frac{F(t,T)}{S(t)}\right) + r \qquad [\ 4]$$

It is often more useful and indeed intuitively appealing for practitioners to determine the convenience yield in terms of a monetary value per unit of commodity which requires the formulation in [4] to be modified to that in [5]:

$$F(t,T) = (S(t) - CY)\exp[(r+c)(T-t)] \qquad [\ 5]$$

Table 2 shows how this spot/futures equilibrium relationship for a real asset, as modified by the addition of a convenience yield expressed in monetary terms, is constructed with an arbitrage portfolio.

Table 2: Classical arbitrage portfolio with convenience yield

Operations	**Debits and credits in the portfolio at time t**	**Debits and credits in the portfolio at time T**
Buy the physical asset (the real cost is now S – CY)	- S(t) + CY	
Borrow the amount to pay the physical asset and pay the interest to the bank and the commodity insurance at time T	S(t) – CY	[-S(t) + CY]*[exp(r+c)(T-t)]
Sell a futures contract whose maturity is (T-t) at time t and deliver the physical asset at time T	Value(F(t,T))=0*	F(t,T)
Imbalance	0	0

The convenience yield *CY* now expressed in monetary terms may be directly calculated by rearranging [5] as follows:

$$CY = S(t) - F(t,T)\exp[-(r+c)(T-t)] \qquad [\ 6]$$

The relationship in [6] is the most commonly used method to calculate the convenience yield in commodity markets because of its theoretical robustness and in the remainder of the paper this formulation will be referred to as the *classical convenience yield*.

Introducing the convenience yield allows the classical theory of storage to hold for real assets but it is a stochastic variable which means that the spot/futures relationship, as expressed in [3] and [5] above, is unstable. Therefore the convenience yield formulation derived in [4] and [6] may only be used to calculate the convenience yield for the current period (t) from currently observable prices and may not be used to forecast what the convenience yield will be in future periods ($t_{1..n}$)

1.2. Literature Review

The literature presents four differing explanations for the occurrence of the convenience yield.[2] First, Kaldor (1939) and Brennan (1958) justify the convenience yield in a backwardated market by saying that stockholding allows economic agents to cope with demand without being subjected to the risk and cost of waiting for a future delivery. Working (1949) further states that fixed costs incurred in the storage activity are insufficiently high to incite producers to sell a commodity when the market is in certain conditions of scarcity and that it is this that induces a backwardation. Weymar (1968) observes that the convenience yield is a decreasing function with respect to the level of stocks and that stocks are retained in a backwardated market because the production, storage and transportation activities are insufficiently flexible to be able to adapt instantaneously to changes in supply and demand. Finally, Williams & Wright (1989) assert that stockholding is the best way to minimise 'transformation' costs (i.e. transport, processing, and marketing).

The literature therefore suggests that convenience yield is both a measure of the benefit accruing to the owner from stock holding in a world of uncertainty and also the rigidity cost attached to production, storage, transport and transformation of that commodity. In practical terms it allows the industry to avoid breaching contracts, shutting production facilities or incurring additional shipping costs (e.g. from delays).

Though the convenience yield is most commonly modelled using the modified theory of storage, as described above, the analysis is sometimes refined with the use of option pricing theory such as that proposed by Milonas & Thomadakis (1997). A brief discussion of their approach as well as the recent interesting approximation for convenience yield derived by Heaney (2002) is presented in the remainder of this section. Models such as Gibson and Schwartz (1990) and Schwartz (1997), which represent the convenience yield as a stochastic variable, will not be discussed. Although they are useful as predictors of the term structure of prices they do not provide any economic understanding of the underlying drivers of the convenience yield.

[2] For more information on these four explanations; see Delphine Lautier: "Trois modèles de structure par terme des prix du pétrole: une comparaison". *Cahier de recherche du CEREG* No.9907, Sept. 1999 (Université de Paris Dauphine).

1.3. Milonas and Thomadakis' Model from Option Pricing Theory

An alternative to the classical approach by Milonas and Thomadakis (1997) presents the convenience yield as a call option. They use the example of a crop cycle which begins with production decisions and ends up with the output being produced on the harvest date. The intervening period contains multiple dates on which the producer may choose to exercise an option to sell the physical commodity produced in the last harvest or store it in order to sell it at a later date. The futures price $F(0,1)$ is expressed as follows:

$$F(0,1) = S(0) + C(0,1) - V(0,1) \qquad [7]$$

The current spot price is $S(0)$, while V is the convenience yield of carrying stocks over the period $[0,1]$ and C is the cost of carry. At time Zi, the producer may decide to either keep the inventories until t=1 or to sell them. He will sell the commodity only if $S(Zi) - F(Zi,1) + C(Zi,1) > 0$. Then $V(0,1)$ could be thought of as the value of the contingent claim whose payoff is: $Max\ (0,\ S(Zi)-F(Zi,1) + C(Zi,1))$ in Zi. It is then the same expression as the payoff of a call option with a strike price equal to $F(Zi,1)$. The call option value is the value of the convenience yield and the strike price is a stochastic process. Indeed, from date 0 we do not know what the futures prices in date Zi will be. The best known option valuation model that matches this situation is Fischer's model (1978). The main finding of the Milonas and Thomadakis' study is that convenience yield exhibits many of the characteristics of a call option. Moreover, they show that as the crop cycle advances (i.e time since the last harvest increases and time to the next harvest decreases) the probability of stocks falling to zero increases. As a result the convenience yield will tend to increase through time.

1.4. Heaney's Model

Heaney (2000)[3] proposes a new model of convenience yield that relies for its originality on the relaxation of a key assumption of the arbitrage portfolio underlying the classical theory of storage – which is that a trader holding a stock of physical commodity will

[3] We are grateful to Doctor Richard Heaney for helpful comments related to his article Heaney (2003).

hold a hedged cash and carry position until the maturity date of the offsetting futures contract. In reality, instead of buying and storing the commodity, traders always have an option to sell the commodity at a date t and investing the amount earned at the risk-free rate of return and then buy the asset back at date T (i.e. the maturity date of the futures contract) as a spot commodity.

Heaney's assumption is that traders exhibit perfect price forecasting ability and thus will sell at the maximum price named *S(t*)* where t^* is found by maximising *S(t)* over the period *[0,T]* for $(t,t^*) \in [0,T]*[0,T]$. The maximum price over the period compounded to futures contract maturity is defined as:

$$M_T = \max_{0 \le t \le T} \{\exp[r(T-t)S_t]\}. \qquad [8]$$

That is, the amount earned in T after selling the commodity in t^* and reinvesting the money at the risk free rate of return over the period *[t,T]*. Then, the strategy consists of buying the asset back in T so the value of this trading strategy is:

$$TS(S_t,T) = [E(M_t) - E(S_T)]e^{-r(T-t)} \qquad [9]$$

Longstaff (1995) provides a solution by which an assessment of this strategy may be made in which s is the spot price volatility:

$$TS(S_t,T) = S_t \left\{ \left[2 + \frac{s^2(T-t)}{2}\right] N\left[\frac{s^2(T-t)}{2}\right] + \sqrt{\frac{s^2(T-t)}{2p}} \exp\left[\frac{-s^2(T-t)}{8}\right] - 1 \right\} \qquad [10]$$

-

Moreover, Heaney explains that 'the value of profitable trading opportunities available to the underlying asset position relative to the value of profitable trading opportunities available to the futures contract position generates the approximation for the convenience yield'. Then the convenience yield over the period [0,T] is equal to:

$$CY_{0T} = TS(S_t,T) - TS(F_t,T) \qquad [11]$$

The primary contribution of Heaney is that he provides a closed form approximation for convenience yield. This approximation requires estimates of futures price volatility, spot price volatility and time to futures contract maturity. The key point is that this approximation applies after traditional costs of carry elements such as interest rate have been accounted for and it focuses on the time from now until futures contract maturity.

2. THE NORTH SEA CRUDE OIL MARKET AND THE DELIVERY ISSUE

Unfortunately, none of the three approaches to modelling convenience yield, described in the previous section, can be applied successfully to the Brent crude oil market. This is because there is a crucial logistical issue in this particular market (and that for other North Sea oil grades) that changes the basis upon which traders formulate and execute their trading strategies. That is that they are all waterborne grades traded under FOB contracts.[4]

2.1. The Brent Complex

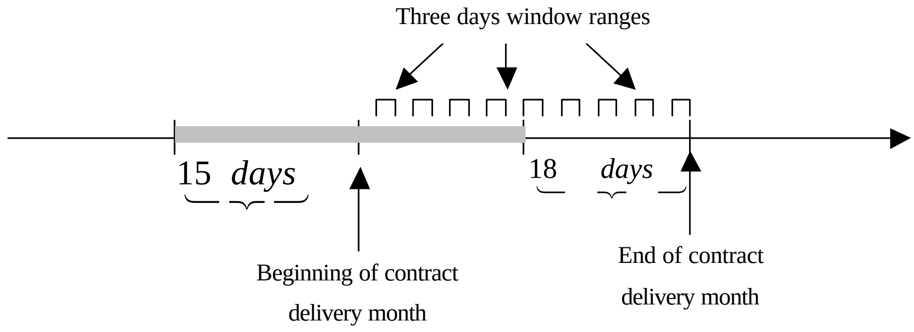
The market for Brent crude is really a complex of four closely related markets for different commodities linked to the underlying spot market value of Brent crude oil and governed by distinct but interrelated contractual terms:

i. Brent forward contract
ii. Dated Brent (now replaced by BFO) contract
iii. IPE Brent futures contract
iv. Brent swaps and options

The trading of Brent crude on a forward basis became established in 1981 and gradually extended out to delivery dates up to 2 years ahead. It had a classic forward contract structure with standard 500,000 barrel parcels being bought and sold on the basis of a pre-specified future (calendar) delivery month. Under the contract, the seller was obliged to nominate a parcel of physical Brent crude to the buyer with a 3-day lay-day loading date range falling in the calendar month in question with a minimum of 15 days notice between nomination date and lay-day range. Nominations therefore always began 15 days before the beginning of the relevant delivery month and ended 18 days before the end of the relevant delivery month. The buyer was obligated to charter a vessel capable of arriving for loading during that period. The schematic in Figure 1 explains the nomination process timeline where the grey tint area represents the period during which the seller's nominations took place.

[4] An FOB (Free on Board) contract only requires the seller to pump a specific grade and quantity of oil to the pipe flange of a vessel nominated by the buyer. In the case of Brent, the pre-specified load port is the Sullom Voe terminal in the Shetland Isles situated off the far North Coast of Scotland.

Figure 1: Original Brent forward market 15-day nomination process



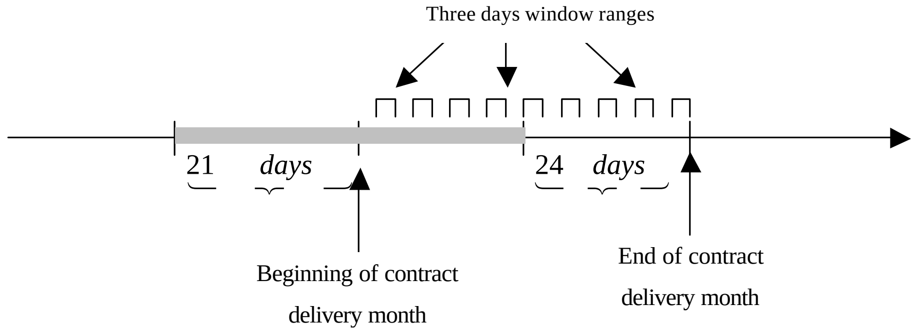
When the period for nominations finished, assuming no counterparty was in default, all forward contracts therefore had a parcel of physical Brent crude oil nominated against them for delivery in the given calendar month with a 3-day loading range attached to it. Once its loading range had been established that parcel of oil became tradable as dated Brent right up to the moment the loading date range began. In a proper sense, spot trading in Brent crude has never existed because logistic constraints impose a delivery delay on the buyer of the crude oil between the day on which a forward contract becomes transformed into a dated Brent contract and the first day of the delivery date range. Typically, dated Brent was therefore traded as a differential to the next available (front month) forward price rather than as an outright price itself.

Despite the apparent success of the Brent crude market and its establishment as the most important crude oil marker in the world, the physical production of Brent began falling in the mid 1990s and by 2002 the volume of dated Brent cargoes fell to such a low level (less than 20 cargoes per month) that it became relatively easy for almost any party to buy up the entire monthly production of Brent and engineer a classic squeeze. That is to say, cause an artificially wide backwardation to be created between the dated Brent price and forward Brent price. As a result dated Brent prices regularly diverged from the ‘spot’ price of other very similar crude oil grades produced in the North Sea and elsewhere in the world. The importance and reliability of Brent as a marker was therefore effectively becoming undermined by a lack of liquidity. A particularly important event highlighted the difficulties of the Brent complex when Saudi Arabia effectively abandoned dated Brent as a price marker in April 2002 in favour of a weighted average of futures prices on the International Petroleum Exchange (see below).

This caused an immediate reaction in the market place and since 10 July 2002 the dated Brent definition has been extended to include two more crude oil grades produced in the North Sea – Oseberg and Forties. These two grades were chosen because they are the closest grades to Brent with regard to quality, price and geographical location of their load terminals. This immediately increased the number of eligible cargoes to over 60 per month and made it impossible to squeeze the market. The economic logic behind the change, according to Platts,[5] is that 'the most competitive grade at the margin will have the greatest degree of influence in the (dated price) assessment' and as Brent has historically been the most competitive grade the new Brent-Forties-Oseberg (BFO) index value would never be far from the former dated Brent price. As well as changing the crude oil grades, the date range over which dated parcels were to be included in the price assessment was changed from 7–15 days before delivery date range to 10–21 days in order to exclude distressed cargoes. In addition, the operational tolerance on the parcel size was narrowed to 1% instead of 5%.

After the BFO's introduction the process of nomination in the Brent forward market was also changed, as shown in Figure 2, with nominations beginning some 21 days, instead of 15 days, before the 3-day loading date range.

Figure 2: The Brent forward market nomination process after BFO introduction



The IPE began trading its Brent futures contract in 1989, which is very similar in character to many other exchange traded commodity futures contracts. The underlying crude oil volumes traded via IPE futures have traditionally been far smaller than forward Brent contracts and are therefore a less important but nonetheless influential

[5] Platts is the most important price reporting agency for crude oil grades around the world and prices published in their daily *Platts Oilgram* are widely used as a basis for pricing many crude oil contracts.

part of the Brent complex. There are three ways to close an open futures position: the exchange for physical (EFP) that allows investors to undertake a swap between a futures position and a forward position, the option to cash settle and exchange for swaps (EFS), or the most popular mechanism which is cash settlement against the IPE Brent Index. This index is calculated daily as an average of the front month forward trades in 21-day BFO cargoes, the second-month trades in BFO cargoes plus/minus a straight average of the spread trades between the first and second months and a straight average of all assessments published in media reports, including Platts.

The most crucial aspect of the IPE contract is that the true underlying (deliverable) asset is not physical Brent crude but what is in effect a short-term forward contract for future delivery of Brent crude because the IPE contract is an FOB cargo contract and not a pipeline contract. That is to say physical delivery is practically impossible due to logistic difficulties. The IPE insists that the Brent futures contract is 'a deliverable contract based on exchange for physicals (EFP) delivery with an option to cash settle' but this is semantics since EFP is more a way of transforming a futures contract into a physical forward contract than delivering physical oil against a futures contract. That is why the vast majority of open futures positions are settled against the IPE Brent index.

The Brent complex is completed by the over the counter (OTC) swaps market where inter-month or inter-crude's swap transactions are effected. These swap contracts have existed since 1991 and the most widely used is a contract for difference (CFD) that allows investors to trade the spread between the dated Brent (BFO) price and the forward price of a given month. This provides a mechanism for hedging the basis risk that buyers and sellers face if the differential between dated Brent and forward prices change. Trading options on various elements of the Brent complex always have been and still are likely to remain a minor component of the overall turnover in the market.

2.2. A Missing Spot Market

In general, a spot market is a market in which commodities are bought and sold for cash and delivered immediately – but not so in the Brent (BFO) crude oil market where once a parcel of oil can no longer be traded in the forward market it may continue to be traded as a dated BFO contract for up to 21 days. Indeed, a very large volume of trading occurs between the end of the nomination process and the loading date of each parcel. In its original form, dated Brent cargoes traded for some considerable time after the date on which they are first nominated as dated Brent (80% in 1991 according to Horsnell and Mabro (1993), page 114, see Table 8.1) with approximately 13 days usually elapsing between the dated Brent contract agreement and the first day of the loading range. Now under the BFO arrangements the average is some 17 days.

In the dated Brent (BFO) market, the commodity is not transportable immediately by the buyer so the dated Brent (BFO) market cannot therefore be regarded as a true spot market though it is frequently referred to as such. Indeed, a commodity that is not transportable immediately from the place where it is delivered has no value as a spot commodity. As the loading date of physical Brent crude bought under a forward contract is subject to a 21-day delay once the forward contract has expired then that parcel of that particular crude oil grade (Brent) nominated under a forward contract has no value as a spot commodity at that location. However, it will have a value on the short-term forward market, called the dated Brent market, which has a maturity date from 3 days up to 21 days before final delivery on to the tanker (ship) that ultimately carries it from the terminal. A true spot market for Brent and other North Sea oil grades is therefore missing.[6]

The position is quite different for WTI crude oil. In this case, there are two delivery points at Cushing (Oklahoma) where NYMEX futures contract deliveries and trading for the mid-continent occur and the second is Midland (Texas). These two delivery points are the hubs where several pipelines join, making them the locations of two spot markets. It is possible to obtain crude oil at either one of them the day after making a spot transaction as long as pipeline capacities permit it. This is very different from the case of a dated Brent (BFO) contract that may only be available for loading some 21 days after a transaction is made.

In conclusion, there are two true spot markets in WTI at Cushing (Oklahoma) and Midland (Texas), whereas in dated Brent there is no spot market at Sullom Voe but

[6] Brent and other North Sea grades are occasionally traded after loading as CIF (Cost-Insurance Freight) cargoes but these trades are so infrequent and for such diverse delivery locations that deriving a proxy spot Brent price from them is not possible.

only a short-term forward market. Thus, if there is no spot market for Brent (BFO) crude oil there must therefore be no true spot price and the dated Brent (BFO) price that is most frequently used as a spot price marker in trading a wide range of other crude oil grades is in reality a short-term forward price.

In the next section we will derive a new spot/futures relationship that takes account of the fact that dated Brent (BFO) trades on a forward contract, rather than as a true spot contract basis, and thereby provide a new method for calculating a true spot price for Brent (BFO) crude oil that currently does not exist.

2.3. The Arbitrage Portfolio of the Futures Seller

In the remainder of this section, IPE futures contract prices are used to derive a new arbitrage pricing model formulation that includes three dates and two periods over which the futures contract delivers a short-term Brent (BFO) forward contract rather than a true spot contract. The variables that are required are *S (t)* representing the dated Brent (BFO) price at date *t*. The difference between the dated Brent (BFO) price, which is actually a forward price, and a true Brent (BFO) spot price is not important in this framework where *f (t,T)* is the forward price of maturity *T* at date *t*, *F (0,T)* the futures price of maturity *T* at date 0, r_0 is the risk free rate of return for the period *[0, T]*, r_t is the risk free rate of return for time period *[t, T]*, and *c* is the storage and insurance cost for the physical commodity. The risk free interest rate can be approximated with the US Treasury bill 3 months and the insurance cost is assumed equal to zero. Hence, in this formulation, the dated Brent (BFO) price becomes equivalent to the true spot Brent (BFO) price plus a correcting factor to take account of the convenience yield and cost of carry. Table 3 summarises the various operations in the construction of the arbitrage portfolio of the futures' seller.

Thus assuming the absence of an arbitrage opportunity:

$$F(0,t) = \frac{S_0 e^{(r_0 + c)T} - f(t,T)}{e^{r_t(T-t)} - e^{r_t(T-t-n)}} \qquad [12]$$

We can see that the buyer will not receive back all the money he paid to the seller at time *t* (i.e. F(0,t) plus interest). There are *n* days of interest that are not paid back to the futures' buyer. The reason for this is a mathematical one. If the seller pays back the buyer for all the money he paid plus the interest then the futures price disappears in the arbitrage equation and we find the classical 'spot/forward' relationship.

Table 3: The arbitrage portfolio for North Sea crude oils

Operations	Debits and credits in portfolio at time 0	Debits and credits in portfolio at time t	Debits and credits in portfolio at time T
Buy the physical asset	- S(0)		
Borrow the amount to pay the physical asset and pay the interests at time T	S(0)		$-S(0)*\exp((r_0+c)*T)$
Sell a futures contract whose maturity is t at time 0 and deliver the physical asset only at time T	Value(F(0,t))=0		
The buyer of the futures contract pays at the expiry of the contract (time t)		F(0,t)	
The seller reinvests F(0,t) at the risk free rate of return		-F(0,t)	$F(0,t)*\exp(r_t*(T-t))$
The buyer pays for the synthetic forward contract he bought on date t		0	f(t,T)
The seller reimburses the buyer for the futures contract payment on date t			$-F(0,t)*\exp(r_t*(T-t-n))$
Imbalance	0	0	0

With the classic arbitrage formula ($F(t,T) = S(t)\exp[(r + c - cy)(T - t)]$) it is impossible to predict the futures price on the basis of the spot price because the marginal convenience yield (cy) must be known. This formula can only be used to find the level of the convenience yield if historic data are available on prices and costs. The formula derived above is therefore an adaptation of the classic arbitrage formula, which can be applied to the Brent (BFO) market.

In Table 4, a nonseasonal ARIMA model classified as an ARIMA (p,d,q) model is presented where *p* is the number of autoregressive terms, *d* is the number of nonseasonal differences, and *q* is the number of lagged forecast errors in the prediction equation. This shows that the classical convenience yield calculated as

$S(t)-F(t,T)e^{-r(T-t)}$) is not a forecastable value since the R-square of the ARIMA model is low and is the reason that some researchers have developed stochastic representations of convenience yield as an alternative approach.

Table 4: ARIMA (2, 1) model for the convenience yield

Model parameter	estimate	Standard error	t-statistic	Probability>|t|
Intercept	-0.00100	0.0010	-0.9950	0.3198
Moving Average, Lag 1	0.97173	0.0085	113.7367	<.0001
Autoregressive, Lag 1	1.31757	0.0233	56.4914	<.0001
Autoregressive, Lag 2	-0.32150	0.0225	-14.2677	<.0001
	R square = 0.19			

However, if the forward price in the formula is replaced with the value of the forward price as expressed in the classical theory of storage then we have:

$$F(O,t)=\frac{S_0\left[e^{(r_0+c)T}-e^{(r_t+c)(T-t)}\right]}{e^{r_t(T-t)}-e^{r_t(T-t-n)}} \qquad [13]$$

The number of days n for which the futures' buyer will not receive the interest that is due to him is highly predictable.

2.4. Relationship between n and the Time to Maturity of the Futures contract

The futures' buyer agrees to pay more when the time to maturity of the futures contract is longer. The further away the expiry date of the front month futures contract is, the higher *n*. However, *n* is not a function of *T-t* (since *T-t* is assumed to be constant[7]) but only a function of *t*. Thus the delay between the futures expiry date to the delivery of the physical commodity does not matter. The buyer only pays for the time to maturity (*t*). Finally, *n* is an increasing function with respect to *T*. And the futures price is a decreasing function with respect to *n* as follows if we assume that r_0=r_2 and *c*=0:

$$\frac{\partial F}{\partial n}=\frac{r_0 e^{r_0(T-t-n)}\left[s_0 e^{r_0(T-t)}-s_0 e^{r_0 T}\right]}{A^2}<0 \qquad [14]$$

[7] It was an average of 13 days under the original dated Brent contract and an average of 17 days under the new dated Brent (BFO) contract but in reality it varies with respect to each and every transaction.

Moreover, we can also see that:

$$\frac{\partial F}{\partial T}=0\ ,\ \frac{\partial n}{\partial T}=0\ ,\ \frac{\partial n}{\partial t}=-1+\frac{e^{r_0(T-t)}(1+S/F)}{e^{r_0(T-t)}(1+S/F)-S/F\,e^{r_0T}}>0\cdot \qquad [15]$$

Actually, the difference between n and t allows for the existence of a convenience yield. This point is decisive and bears explanation.

Suppose we replace n by t in equation [13] and if we consider r_0 equal to r_2 then our model could be expressed as:

$$F(0,t)=\frac{S_0\left(e^{r_0T}-e^{r_0(T-t)}\right)}{e^{r_0(T-t)}-e^{r_0(T-2t)}} \qquad [16]$$

This expression is actually the same as the classical theory of storage relationship, in equation [1] just rewritten in the same notation as: $F(0,t)=S_0e^{r_0t}$.

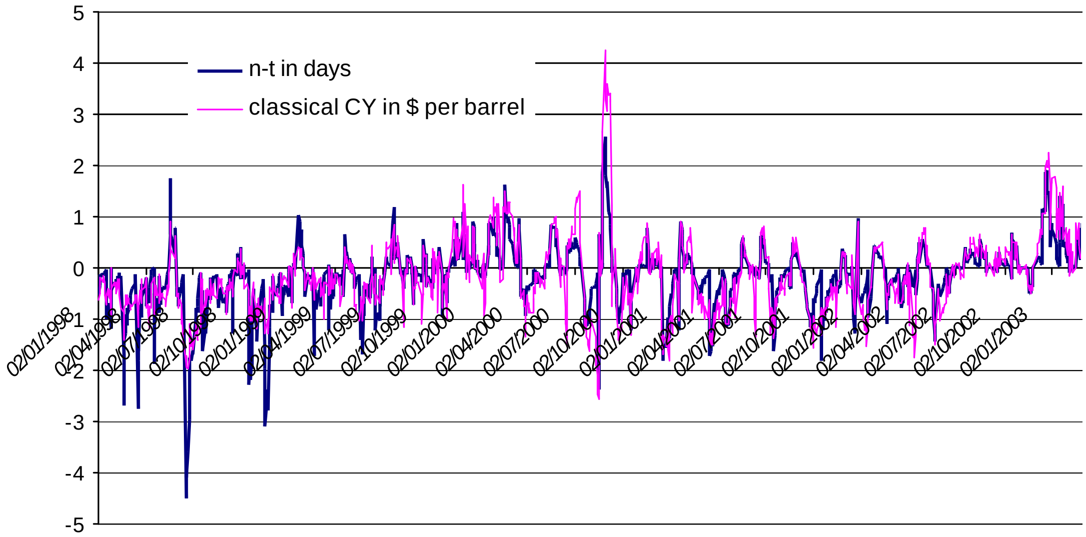
Since n is defined as the number of days over which the buyer will pay interest to obtain the spot price at a future date, our formula is just a modification of the classical theory of storage. If this number is t the buyer agrees to pay the interest over the time to maturity and this is just the cost of carry in the classical theory of storage then our formula is the same as the classical cost of carry relationship. This result occurs because we replaced the forward price by its equivalent in the cost of carry equation that is to say: $S_0e^{r_0t}$ and because we consider that interest rates are equal in both cases. Then the difference between n and the time to maturity of the futures contract is an estimator (in number of days) of the convenience yield. Indeed, if n were equal to t then we would find $F(0,t)=S_0e^{r_0t}$ and so there would be no convenience yield. As we can see in Figure 3, this measure of convenience yield (n-t days) evolves in the same way as the classical formula in [6] as $CY=S(t)-F(t,T)\exp[-(r+c)(T-t)]$.

In conclusion, by modifying the classical theory of storage an alternative formulation of the classical cost of carry relation can be created by introducing n. This relation is as follows:

$$F(O,t) = \frac{S_0\left[e^{(r_0+c)T} - e^{(r_t+c)(T-t)}\right]}{e^{r_t(T-t)} - e^{r_t(T-t-n)}} \qquad [17]$$

From this new term formulation we can therefore derive a new way of expressing convenience yield as discussed in the next section.

Figure 3: Evolution of classical convenience yield and *n-t*



2.5. A New Expression for the Convenience Yield

Reverting to our original formula:

$$F(0,t) = \frac{S_0 e^{(r_0+c)T} - f(t,T)}{e^{r_t(T-t)} - e^{r_t(T-t-n)}} \qquad [18]$$

Obviously this does not provide a complete picture of the spot/futures relationship[8] in the Brent (BFO) market but it will be helpful in calculating a true convenience yield. At this point, it is very important to understand that the classical theory of storage still

[8] It gives a Futures/Forward/Spot relationship.

holds even if there is no storage in the market because the adjustment of the spot/futures relationship formula is made through the convenience yield. In which case, the convenience yield will be very high. In fact, in the oil market in general, the occurrence of a zero stock scenario never occurs for precisely this reason as the holding of some stocks is always necessary for companies to continue running their operations smoothly so as to meet their contractual obligations. Just-in-time stock management does not weaken this argument it just changes the amount of stock held.

Given the relationship in [18] a new way of calculating the convenience yield can be pursued; let us denote the convenience yield as CY:

$$F(0,t) = \frac{\left(S_0 - CY\right)\left(e^{r_0 T} - e^{r_0 (T-t)}\right)}{e^{r_0 (T-t)} - e^{r_0 (T-t-n)}} \qquad [19]$$

If we assume that $t = n$ then we find the convenience yield as calculated in the classical cost of carry relation. In that case, we obviously come back to the departure point. Indeed, we have:

$$CY = \frac{F(0,t)\left(e^{r_0 (T-t)} - e^{r_0 (T-2t)}\right)}{e^{r_0 T} - e^{r_0 (T-t)}} + S_0 = S_0 - F(0,t)e^{-r_0 t} = \text{Classical CY} \qquad [20]$$

However, if we set the forward price as it is in the formula [6] we will find a more interesting and fundamental result in which the forward price reflects the specific nature of the dated Brent (BFO) market. Let us replace r_0 and r_t by r in equation [6] then we have:

$$F(0,t) = \frac{S_0 e^{rT} - f(t,T)}{e^{r(T-t)} - e^{r(T-t-n)}} \qquad [21]$$

To calculate the convenience yield we first need to look at the values for *n* and observe the deviation from the time to maturity: *t* as follows:

$$n_h = (T - t) - \frac{1}{r_h} \ln\left(\frac{-S_h e^{rT} + f(h+t,T)}{F(h,t)} + e^{r(T-t)} \right) \qquad [21]$$

Note that the spot and the futures prices are taken at date *h* whereas the forward price is taken at time *h* + *t* (i.e. where *t* is the time to maturity of the futures contract). Calculating the difference between n and t, which is denoted *n'* and then the equivalent of *n'* in dollars: convenience yield.

2.6. Testing the New Formulation

In this section we will test the new formulation for convenience yield as expressed in equation [21] derived above by using daily time series data from the Brent complex for the period January 1998–February 2003. Settlement prices for the 1st month Brent futures contract on the IPE, the 15-day dated Brent and 21-day BFO price as published by Platts are used. The approximation for the convenience yield is given by equation [10]:

$$CY_h = \left[r_h * (T - 2t) - \ln\left(\frac{-S_h e^{rT} + F(h,T) - \sum_{i=h}^{T-1} \left(F(i+1) - F(i)\right)\left(\frac{1}{p(i+1)} - 1 \right)}{F(h,t)} + e^{r(T-t)} \right) \right] * F(h,t) \qquad [22]$$

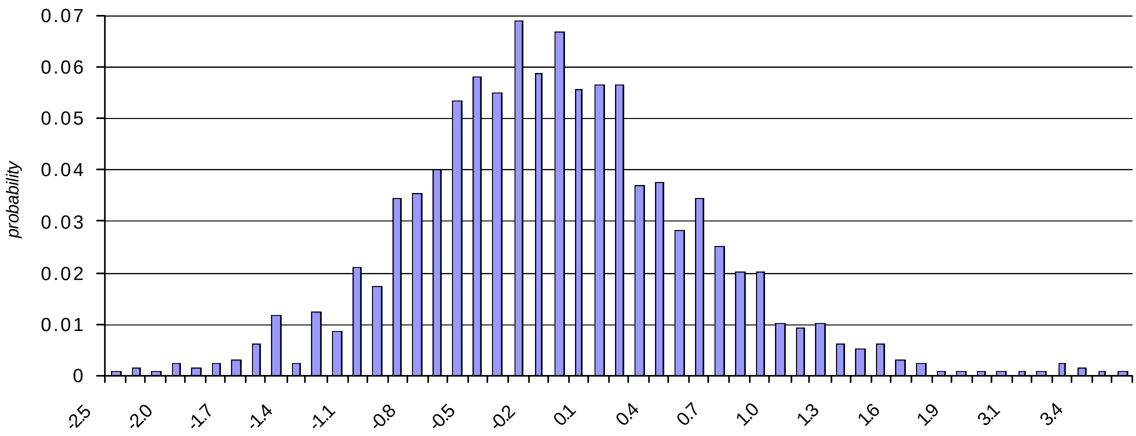
Annex 1 presents the calculation of the convenience yield with this method where the futures price that is in the numerator inside the *ln* operator is calculated as follows:

$$F = \frac{17}{30} * firstfuturesposition + \frac{13}{30} * 2ndfuturesposition$$

The futures price that is in the denominator is the IPE front month futures price.

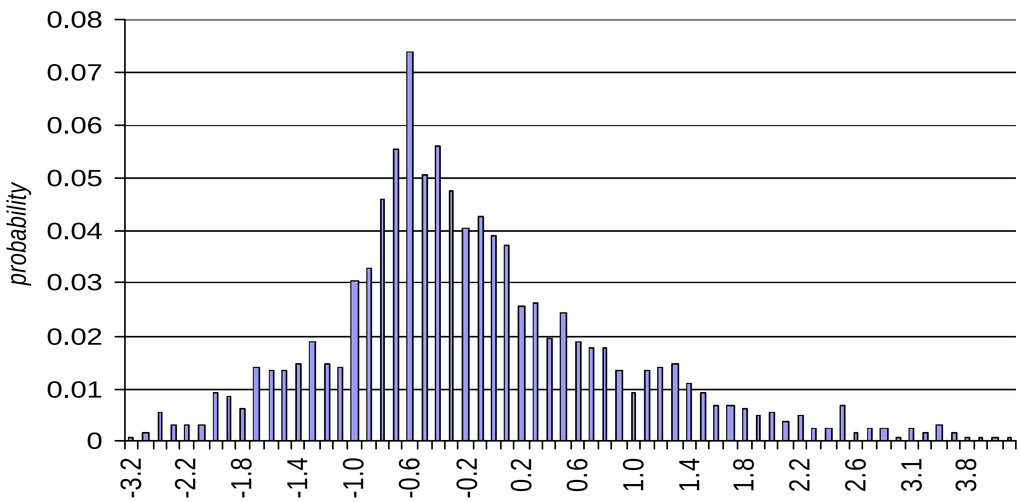
Figures 4 and 5 show the empirical density functions of convenience yield worked out with the classical method and the new formulation. The analysis in Table 5 shows that the range of values taken by the convenience yield is almost the same for both formulas. The kurtosis is lower with the new formula, which means that percentage of values around the mean is lower than with the classical relationship. Moreover, the standard deviation is higher (two times) with our formula, that is to say the convenience yield tends to be more volatile. In the same vein, the convenience yield of dated Brent price is more volatile with the new formula as the latter also includes the forward price. So, the link between spot price and convenience yield is weaker in the new formulation.

Figure 4: Density function of convenience yield from classical formulation



Source: own calculations

Figure 5: Density function of convenience yield from new formulation



Source: own calculations

Table 5: Features of the convenience yield calculated with both formulas

	CY		**% CY/spot price**	
	new	classical	new	classical
average	-0.14	-0.106	-1.498	2.835
standard deviation	***1.05***	***0.735***	***4.667***	***2.632***
kurtosis	***1.39***	***3.402***	***0.319***	***7.144***
skewness	0.83	0.801	***-0.045***	***2.132***

In the next section the new convenience yield expression derived above will be applied to explaining the Brent/WTI price differential (spread).

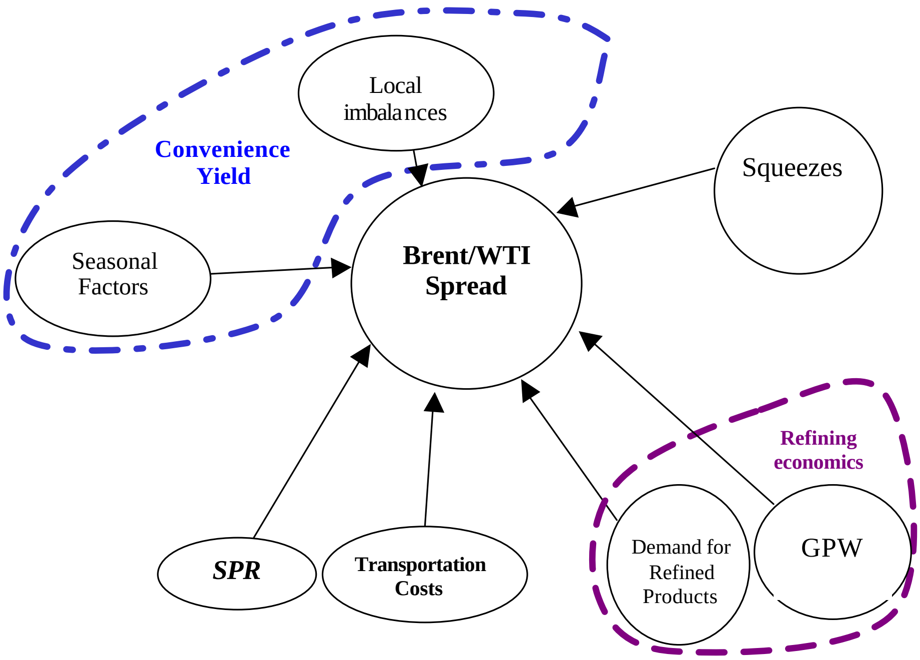
3. EXPLAINING THE BRENT/WTI SPREAD USING CONVENIENCE YIELD

Arbitrage trade between the North West European crude oil market and the North American crude oil market is underpinned by the physical transportation of Brent crude and other North Sea crude oil grades to the US Gulf Coast refineries. Analysis and understanding the behaviour of the price differential (spread) between the WTI price and the Brent price is therefore a central empirical issue in the daily functioning of the global oil market.

3.1. Determinants of the Brent/WTI spread

Figure 6 shows there are six key determinants of the Brent/WTI spread but only one factor is structural: the transportation costs.

Figure 6: Determinants of the Brent/WTI spread



Indeed, the transportation costs are normally the only variable that should differentiate in the long term the prices of the two crude oils since they have close specifications as shown in Table 6. Among the factors listed above the gross product worth (GPW) reflects the value of the specific grade or blend in terms of product yields.

Table 6: WTI and Brent specifications

	Brent	WTI
Gravity (API)	38.5	39.6
Barrels per metric tonne	7.522	7.615
Viscosity (centistokes at 40 C)	3.74	3.73
Sulphur content (% weight)	0.36	0.24
Pour point (Temp. C)	-12	-21

Source: Petroleum Intelligence Weekly, crude oil handbook

Different temporary supply/demand phenomena occur in the oil market and affect the WTI/Brent spread. For instance, demand for crude oil and the level of crude oil inventories in the USA often diverge for a temporary period, then WTI prices change but not Brent prices. Moreover, change in demand and supply for oil is not constant throughout the year; there are sometimes supply disruptions and demand is seasonally affected in different ways in the US and European markets so prices for the two marker crude oils are differentially affected by events that occur in the two markets.

3.2. Classical and New Convenience Yields in Explaining the Brent/WTI Spread

The convenience yield is an explanatory variable of the WTI/Brent spread since it is an expression of the factors enumerated above. In its classical form the convenience yield is defined as an incremental value of spot prices over futures prices after accounting for carrying costs. A comparison of the WTI and Brent convenience yields, calculated with the classical formula is shown in Table 7 for the period January 1995–February 2000.

Table 7: Daily convenience yields from 03/01/1995 to 28/02/2000 ($ per barrel)

	WTI		Brent	
months from maturity	mean	st. dev.	mean	st. dev.
1	0.08	0.26	-0.05	0.51
2	0.28	0.61	0.15	0.82
3	0.47	0.94	0.32	1.12

Source: Own calculations

The regression analysis we present below aims to explain the WTI/Brent spread through convenience yields attached to North Sea oil grades and WTI. We undertake two regressions: the first uses the classical convenience yield formula while the second uses the new formulation described in the previous section. In all three cases the convenience yield that refers to the WTI market is calculated with the classical formula because we know that there is true WTI spot market and no need for the new formulation to be applied to it. The simple regression equation [11] is used for that purpose.

$$\Delta\left(WTI_{spot} - datedBrent\right) = a + b\Delta\left(CY_{Brent}\right) + g\Delta\left(CY_{WTI}\right) \quad [23]$$

Table 8 presents the results when the classical convenience yield is used to represent the North Sea oil market convenience yield and Table 9 shows the results of the regression when the new formulation is used.

Table 8: Results of the regression with the classical convenience yield

Variable	Parameter estimate	t-value
Intercept	0.00138	0.13
CY Brent (CLASSICAL)	-0.06840*	-2.8
CY WTI (CLASSICAL)	0.57766*	14.96
	R-square: 0.1565	Durbin-Watson: 2.071

**Statistically significant at 1%*

Table 9: Results of the regression with the new formulation

Variable	Parameter estimate	t-value
Intercept	0.00194	0.2
CY Brent (NEW FORMULATION)	-0.40343*	-17.96
CY WTI (CLASSICAL)	0.60493*	17.52
	R-square: 0.3232	Durbin-Watson: 2.099

**Statistically significant at 1%*

We observe that the new convenience yield as expressed with the new formulation is a better explanatory variable as indicated by the increase in t-value from -2.8 to -17.96 for the CY Brent independent variable. Note also that the parameter itself increases in magnitude from being virtually zero under the classical formulation. In summary, it seems that our new formulation for the convenience yield in Brent (BFO) is a far better explanatory variable for explaining the Brent/WTI spread than the classical formulation. From an economic point of view it is therefore a better representation of the benefit that refiners on both sides of the Atlantic gain from the possession of spot physical Brent crude oil (or other similar North Sea grades) rather than what is in effect a short term forward-dated Brent (BFO) contract.

4. CONCLUSION

The relationship between futures, forward and spot prices adapted to the North Sea crude oil market allows an expression to be derived for the convenience yield whose calculation is based upon the difference between two periods of time. The first is the number of days over which the futures buyer compensates the seller for holding stocks before delivery. This is the adjustment variable of the model. The second period of time is the time to maturity of the futures contract. These two periods are equal in the framework of the classical theory of storage when the convenience yield is zero: this means that the futures' buyer only pays enough to cover storage costs over the exact time to maturity of the futures contract. Thus, it is the difference between the two periods of time that allows the existence of a convenience yield. The new convenience yield formulation presented in this paper takes into account the specific nature of the North Sea oil market and especially trade in Brent (BFO), in such a way that the real underlying asset of a futures contract is recognised as a short-term forward contract called dated Brent (BFO) and not a physical spot asset.

Empirical observations show that our new formulation fits historic market conditions; in other words, it reflects reality well in both tight and slack crude oil markets. It also appears that the new formulation for convenience yield has greater explanatory power in respect of the WTI/Brent (BFO) spread than the classical convenience yield.

It remains an open question whether this new approximation for the convenience yield is itself easier to explain in terms of the underlying fundamentals of the market. For example, it would be interesting to see whether explanatory variables such as stocks, freight rates, total supply, total demand, are more statistically significant in econometric models with the new formulation as the dependent variable than is typically the case when the classical convenience yield is used as the dependent variable. We hope that this new formulation of the convenience yield therefore leads others to pursue further empirical research in this area.

ANNEX 1

This annex provides a spreadsheet formulation to get an approximation of the convenience yield. The forward price is assessed directly using the futures price without using equation [9] as follows where F1 is the front month futures price and F2 is the second position price:

$$F = \frac{17}{30} * \mathit{firstfuturesposition} + \frac{13}{30} * 2\mathit{ndfuturesposition}$$

Then the convenience yield formulation becomes:

$$CY_h = \left[r_h * (T - 2t) - \ln\left(\frac{-S_h e^{rT} + 17/30 * F_1 + 13/30 * F_2}{F(h,t)} + e^{r(T-t)} \right) \right] * F(h,t)$$

=IPE expiry date – current day
= 15* – DAY(A5) = 7

	A	B	C	D	E	F	G	H	I	J
	Date	S(t)	r	t	T	First position (futures)	Second position (futures)	n	CY $/barrel	Approx. forward
1	02/01/1998	15.77	0.0001478	13	26	16.42	16.58	-265.63	-0.68	16.49
2	05/01/1998	15.29	0.0001453	10	23	15.79	15.98	-227.32	-0.54	15.87
3	06/01/1998	15.48	0.000145	9	22	15.67	15.83	-91.58	-0.23	15.74
4	07/01/1998	15.33	0.0001453	8	21	15.61	15.68	-115.00	-0.28	15.64
5	08/01/1998	15.47	0.0001425	7	20	15.72	15.8	-106.36	-0.25	15.75
6	09/01/1998	15.33	0.0001403	6	19	15.5	15.57	-72.81	-0.17	15.53
7	12/01/1998	15.07	0.0001422	3	16	15.33	15.42	-120.14	-0.27	15.37
8	13/01/1998	15.08	0.0001436	2	15	15.38	15.4	-123.69	-0.28	15.39
9	14/01/1998	14.81	0.0001439	1	14	15.35	15.36	-228.69	-0.51	15.35
10	15/01/1998	14.75	0.0001425	0	13	15.44	15.21	-250.87	-0.55	15.34
11	16/01/1998	14.89	0.0001431	27	40	15.47	15.62	-247.92	-0.61	15.54
12	20/01/1998	14.95	0.0001428	23	36	15.4	15.54	-194.13	-0.48	15.46
13	21/01/1998	14.67	0.0001425	22	35	15.26	15.4	-260.12	-0.61	15.32
14	22/01/1998	14.35	0.0001425	21	34	15	15.11	-287.28	-0.66	15.05
15	23/01/1998	14.26	0.0001436	20	33	14.74	14.88	-219.50	-0.51	14.80
16	26/01/1998	14.79	0.000145	17	30	15.77	15.83	-399.19	-0.95	15.80

=D15+13

=(17/30)*F +(13/30)*G

=(H7-D7)*C7*F7

=13-(1/C3)*LN ((J3-B3*EXP(C3*E3))/F3+EXP(C3*13))

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