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The Recycling of Oil Revenues: Some Theoretical Aspects

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ABSTRACT

We investigate the stability properties of a non-tatonnement price and monetary adjustment mechanism involving an oil exporting and an oil importing country. The distinguishing characteristic of the paper is that it attempts to bring together some elements of the disequilibrium theory of exhaustible resources and the monetary approach to the balance of payments using a Hicksian, temporary equilibrium framework. We show that a sufficient condition for stability is that the marginal propensity to consume out of an increase in money wealth is the same in both countries.

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INRODUCTION

The purpose of this paper is to investigate the stability properties of a non-tâtonnement price and a monetary adjustment mechanism involving two countries: one oil-exporting and one oil-importing. Its distinguishing characteristic is that it brings together some elements of the theory of exhaustible resources and the modern balance-of-payments theory using a Hicksian, temporary equilibrium framework.

Although some authors have tried to introduce elements of international trade into the theory of exhaustible resources (see Dasgupta, Eastwood and Heal, 1978; Chichilnisky, Heal and Sepahban, 1983) it has remained an equilibrium theory with very little to offer to macroeconomic problems. Its main assumptions, that spot and asset markets clear instantaneously and that a complete set of forward markets exists, make it inappropriate for the analysis of "short-term" phenomena. The monetary aspect of international transactions is ignored, and issues related to the recycling of oil revenues and the stability of the international exchange process are left untouched. On the other hand, balance-of-payments theories applied to the problem of recycling of oil revenues have paid no attention to the fact that oil is not only a commodity but an asset as well. To bring these two theories together we have used a Hicksian, temporary equilibrium framework similar to that developed by Dixit and Norman (1980) for the theory of the balance-of-payments. In this framework oil is treated as an asset. We use a formulation due to Heal (1975) according to which oil supply is governed by the difference between the current and the expected oil price.

Since the work contained in this paper stems from two independent disciplines, it is hoped that it provides some contribution to both of them. First, it might be considered as an extension to a two-country world of the dynamics of fixed-price equilibria and monetary adjustment. The only attempt at a systematic stability analysis along these lines is that of Liviatan (1979) who used a single, small open economy framework introduced by Dixit (1978). Introducing a two-country world makes a substantial difference to the model and the results. Second, it might be considered as an attempt to introduce balance-of-payments considerations into the work of Heal (1975; 1981) and Stournaras (1984) concerned with disequilibrium and stability in exhaustible resource markets.

The two-country two-period framework that we will consider in this paper has certain similarities to that of Marion and Svensson (1984). However, they consider a two-period economy with a perfect world capital market and a forward oil market while in our economy only spot commodity and money markets are open. In addition, they do not treat oil supply as an asset management decision and ignore exhaustibility.

The rest of the paper is organised as follows. In the first two sections we describe the economy and derive the temporary, international equilibrium conditions. In the third section we set up an adjustment process, designed to capture the function of a decentralized economy where spot and asset markets do not clear instantaneously, whose stationary point is the temporary equilibrium. In the fourth section we derive some preliminary results. In the last section we study the (local) stability properties of this process and examine how sensitive the results are to the various assumptions. An interesting point to notice might be the similarity of these assumptions to those derived for the analysis of the familiar "transfer" problem in international trade.

DESCRIPTION OF THE ECONOMY

We will assume that the world consists of two countries, or two groups of countries. The oil-exporting country, OPEC, and the oil-importing one, OECD. The economy will contain two goods and one financial asset. One of the two goods, oil, is an exhaustible resource owned by OPEC. It will be used as a factor of production in OECD to produce a consumption good d. This will be consumed by both OPEC and OECD. OPEC's consumption will be identical to its imports and its exports will be identical to its oil production.

The economy's financial asset will be fiat money. It will yield a zero rate of return and its total world supply will be fixed, $\bar{M}$. In addition, it will be the numeraire commodity and will be used both as a medium of exchange and a store of value. (The intertemporal structure of the economy will be specified below). We will assume that the exchange rate is fixed (pegged) and normalized to unity. Hence, if OPEC is allocated $\bar{m}$ units of money, OECD's allocation will be $\bar{M} - \bar{m}$.

The intertemporal structure of the economy will resemble a Hicksian, two-period world. (See, among others, Dixit and Norman, 1980, chapters 7, 8 and 9.) There will be no forward markets in the first period. Only spot markets and the money market are open. During the first period traders may form expectations about future prices. By assumption these will be point expectations and will be held with certainty. The formation of expectations is an important feature of the model. Since oil is a commodity which can be sold now or kept in the ground in order to be sold in the second-period (the "future"), its current supply will be governed by the difference between the current price and the price expected for the second period.

In the next section we derive the various demand and supply functions and the first period (temporary or short-run) market clearing conditions.

TEMPORARY EQUILIBRIUM

The two countries' excess demand functions will be derived by the standard utility maximization procedure. OECD's demand for oil, demand for money, demand for and supply of the consumption commodity will be given by the solution to:

$$\text{Max} \quad U[d^1, \frac{M}{f}] \tag{1}$$

Subject to: $$qd^1 + M = \Pi + (\bar{M}-\bar{m}), \tag{2}$$

where Π is the solution to:

$$\text{Max } \Pi = qY - pC \tag{3}$$

Subject to: $$Y = F(C) \tag{4}$$

U is a well-behaved utility function and depends on consumption d^1 and real money balances M/f. f is a linear homogeneous function of the consumption price q^1. Real money balances represent future expenditure (savings). (2) is the budget constraint. $\bar{M}-\bar{m}$ is OECD's initial allocation of money and Π is the net value of OECD's production activity from transforming crude oil, which is bought at a price p, into the consumption commodity d. Π is maximized[2] subject to the strictly concave production function (4). It might be noticed that (2) provides the link between OECD's rate of money accumulation and its trade surplus:

$$M - (\bar{M}-\bar{m}) = \Pi - qd^1 = q(Y-d^1) - pC \tag{5}$$

The solution to the above problem gives a set of supply and demand functions:[3]

$$Y = Y(p,q) \qquad (6)$$

$$C = C(p,q) \qquad (7)$$

$$d^1 = d^1[q, \Pi(p,q), \bar{M}-\bar{m}] = d^1(q,p,\bar{M}-\bar{m}) \qquad (8)$$

(6) and (7) are derived from (3) subject to (4). (8) is derived from (1) subject to (2), where $\Pi = \Pi(q,p)$ is given by (3) and (4).

Similarly, OPEC's demand for consumption, supply of oil and demand for money will be given by the solution to:

$$\underset{f}{\text{Max}} \; W[d^2, \underline{m}] \qquad (9)$$

Subject to: $qd^2 + m = \bar{m} + pS^*$ (10)

where S* solves the problem:

$$\underset{S}{\text{Max}} \; pS + e(\bar{R} - S) \qquad (11)$$

(9) and (10) are similar to (1) and (2). S* is current oil supply and is chosen to maximize the net value of oil over the two periods. By assumption, extraction costs are zero and total oil reserves are $\bar{R}$. e is the expected oil price for the second period; we will explain how e is formed later.[4]

From (11) we obtain the present value profit maximization condition:[5]

$$p=e \qquad (12)$$

From (9) and (10) we obtain:

$$d^2 = d^2(q,T,\bar{m}) = d^2(q,p,\bar{m}) \qquad (13)$$

where T is defined as $T = pS^*$.

Equation (10) provides the link between OPEC's rate of money accumulation and its trade surplus:

$$m - \bar{m} = pS - qd^2 \qquad (14)$$

We are now ready to produce the conditions required for the first-period equilibrium. These are:

$$H = d^1 (q,d,\bar{M} - \bar{m}) + d^2(q,p,\bar{m}) - Y(q,p) = 0 \qquad (15)$$

$$E = C(p,q) - S^* = 0 \qquad (16)$$

$$pS^* - qd^2 = 0 \qquad (17)$$

$$p = e \qquad (18)$$

Equation (15) is the market equilibrium condition for the consumption commodity d. Equation (16) is the condition for equilibrium in the oil spot market. Equation (17) is the condition for equilibrium in the money market (we will explain this below), and (18) is the condition which guarantees that the owner of oil (OPEC) is indifferent between selling oil now or in the future.

Equation (17), which requires that OPEC is in a balance-of-payments equilibrium, is also the condition for money market equilibrium and, since the world money supply is fixed, the condition that OECD is in a balance-of-payments equilibrium. This is easy to show: using equations (14), (15), (16) and (17), equation (5) yields:

$$M - (\bar{M} - \bar{m}) = qd^2 - pS^* = \bar{m} - m = 0 \qquad (19)$$

Hence:

$$M + m = \bar{M} \qquad (20)$$

which says that overall money demand is equal to money supply.

We have not included in the above system of equilibrium conditions an equation explaining how e is formed. If expectations are consistent with the intertemporal structure of the model (ie if we postulate rational expectations), e must solve the following equation:

$$\hat{E} = C(p,q) + \hat{C}(e,f) - \bar{R} = 0 \qquad (21)$$

where f is the same index as in (1), (9) (a linear homogeneous function of q) representing the expected price of good d. $\hat{E}$ can be interpreted as an estimate of the future excess demand for oil function.[6] Hence, expectations will be rational if e solves (21).

However, expectations may be formed in a different way. For instance they may be formed adaptively or by any other rule of thumb. In the dynamic analysis that follows we simply assume an expectations formation function e = e(.). In order to derive stability results we will need to impose certain conditions. In fact we will see that if e(.) is such that e solves (21), some interesting results can be derived.

NON-INSTANTANEOUS MARKET CLEARING: ON PRICE AND ASSET ADJUSTMENT

A natural question to ask is how the equilibrium described by the system of equations (15) to (18) is achieved. Or, put differently, suppose that the economy starts from an arbitrary point where some or all of these equations are not satisfied. In addition, let us postulate that adjustment occurs according to a certain dynamic process. Will this process converge to equilibrium?

In this and the next section we will specify such an adjustment process and study its stability properties. To simplify the analysis we will assume that the market for the consumption commodity clears instantaneously, ie condition (15) is always satisfied. This is a strong assumption and affects the results. We will comment on it later.

Asset markets and the spot oil market will not clear instantaneously. Non-instantaneous asset market clearing means that the money market is not in equilibrium and that OPEC is not indifferent between selling oil in the first or the second period. Non-instantaneous spot oil market clearing means that oil demand, given by (7), differs from the ex-ante oil supply, $\tilde{S}$. $\tilde{S}$ will be implicitly given by the asset adjustment rule:

$$\dot{\tilde{S}} = b(p - e), \ b > 0, \tag{22}$$

where $\dot{\tilde{S}}$ is the partial derivative of $\tilde{S}$ with respect to time.[7]

(22) is an assumption first used by Heal (1975) and postulates that if the current oil price exceeds (falls short of) the expected future oil price, the owner of oil wishes to increase (reduce) current oil supply. Its meaning is obvious and needs no further comment. The spot oil price will adjust according to the rule:

$$\dot{p} = a\tilde{E}, \ a > 0 \qquad (23)$$

where $\tilde{E}$ is defined as $\tilde{E} = C(q,p) - \tilde{S}$. The function C (q,p) is given by (7) and the variable $\tilde{S}$ is implicitly defined in (22). This rule postulates that the spot oil price adjusts in proportion to the ex-ante excess oil demand. In effect, it captures Arrow's (1959) well-known idea of how spot prices change when markets are in disequilibrium.

Finally, non-instantaneous money market clearing implies a current account surplus (deficit) for either of the two countries. For instance, if OPEC has a trade surplus (deficit) OECD will have a trade deficit (surplus) and money will flow into (out of) OPEC and out of (into) OECD. This monetary adjustment will be described by the rule:

$$\dot{\bar{m}} = B \qquad (24)$$

where B is OPEC's trade surplus. Since we have assumed that the oil market does not clear instantaneously, actual oil exports will be given by the short side of the market and the functional form of B will differ according to whether excess oil demand or supply prevails. If excess demand prevails ($\tilde{E} > 0$) OPEC's trade surplus is:

$$B' = p\tilde{S} - qd^2\ (p,q,\bar{m}) \tag{25}$$

while in the case of excess supply ($\tilde{E} < 0$) the trade surplus is:

$$B'' = pC(p,q) - qd^2(p,q,\bar{m}) \tag{25'}$$

For a given expectations formation function e = e( ), equations (22), (23) and (24) form a system of three differential equations in the three state variables $\tilde{S}$, p and $\bar{m}$. By assumption, q will solve equation (15), ie it will clear the consumption good market instantaneously.

The presence of equation (24) presents some technical problems. Because of regime switching (that is, because the oil market switches from excess demand to excess supply) the function B defined in (25) and (25') is not differentiable everywhere. Although some stability results are available for such systems,[8] no general theory exists. A way out of the problem is to eliminate one of the variables and construct the phase diagram for an equivalent system of two equations in two state variables, hoping that a geometric study of stability will give some definite results.

In what follows we eliminate $\dot{\tilde{S}}$. We will approximate the asset adjustment equation (22) by an ex-ante supply relationship $\tilde{S}(p,e)$. We will assume that the first partial derivatives of $\tilde{S}(\ )$ exist and, when they are evaluated at the temporary equilibrium, their signs are:

$$\tilde{S}_p > 0,\ \tilde{S}_e < 0. \tag{26}$$

In (26) and for the rest of the paper subscripts will denote first partial derivatives. The meaning of (26) is the following. Suppose that the temporary equilibrium values of (p,e) are

(p^*, e^*). In fact, asset market equilibrium requires $p^* = e^*$. An increase in the current oil price Δp keeping e constant at $e^* = p^*$ creates a positive wedge between the current and the expected oil price ($p^* + \Delta p > p^* = e^*$) and, therefore, asset market adjustment; the owner of oil wishes to increase his current supply: $\Delta\tilde{S} > 0$. Similarly, an increase in e, Δe, keeping p constant and equal to $p^* = e^*$ implies $p < e^* + \Delta e$ and, therefore, $\Delta\tilde{S} < 0$; oil is expected to be more profitable if saved now in order to be sold in the future.

The system of differential equations under study will consist of (23) and (24) where $\tilde{S}$ is now the supply relationship $\tilde{S} = \tilde{S}(p,e)$, satisfying (26). In order to derive stability results we will need the partial derivatives of $\tilde{E}$, B' and B'' with respect to the state variables p and $\bar{m}$. This will be the subject of the next section.

SOME PRELIMINARY RESULTS

First, we will derive $\tilde{E}_p$. It is not straightforward that $\tilde{E}_p < 0$ since the consumption good market and the expectations response have to be taken into account. Of course, one might wish to impose some kind of "Diagonal Dominance" condition right from the beginning in order to ensure that the own price effect dominates the cross price (general equilibrium) effect, and hence get $\tilde{E}_p < 0$. Let us see if this can be derived from weaker assumptions.

Since $\tilde{E}$ has been defined as $\tilde{E} = C(p,q) - \tilde{S}(p,e)$ we have:

$$\tilde{E}_p = A - G \tag{27}$$

where A and G are defined as $A = C_p + C_q q_p, G = S_p + S_e e_p$. Applying the implicit function theorem to equation (15) we get $q_p = -H_p/H_q$. Hence, A can be written as:

$$A = C_p + C_q \left[\frac{d^1_p + d^2_p - Y_p}{Y_q - d^1_q - d^2_q} \right] \tag{28}$$

Assuming that d^1_q, $d^2_q < 0$ (ie that OECD's and OPEC's demand for the consumption good functions are well-behaved) and since $C_p<0$, $C_q>0$, $Y_q>0$, $Y_p<0$ (these conditions follow from 3 and 4) it is straightforward to show that a sufficient condition for A to be negative is:

$$d^1_p + d^2_p \leq 0 \tag{28}$$

In words, (28) requires that the change in the overall demand for the consumption good following an increase in the price of oil should be non-positive. This condition will be satisfied if the income effect on OPEC's consumption is equal to or less than the income effect on OECD's consumption.[9] It is easy to see why. From the definition of d^1, see (8), and d^2, see (13), and keeping q constant, condition (28) is the same as:

$$d^1_{\Pi}\Pi_p + d^2_T T_p \leq 0 \tag{29}$$

Evaluating the left hand side of (29) at the temporary equilibrium, $-\Pi_p$ is equal to T_p. Following an increase in the oil price, dp, OPEC's current revenues and OECD's costs increase by Cdp. Hence, (29) is equivalent to:

$$C\,(d^2_T - d^1_{\Pi}) \leq 0 \tag{31}$$

and since $C > 0$, to $d^2_T \leq d^1_{\Pi}$ (32)

From now on we will assume that (32) holds. Hence:

$$A < 0 \tag{33}$$

We now turn to $G = \tilde{S}_p + \tilde{S}_e e_p$. Since $\tilde{S}_p > 0$, $\tilde{S}_e < 0$ (see (26) and the subsequent discussion) a sufficient condition for G to be positive is $e_p < 0$. In fact we can show that this will be the case if expectations about the future oil price are consistent with the future availability of the exhaustible resource, ie if e solves equation (21). Applying the implicit function theorem to (21) we obtain:

$$-e_p = \frac{\hat{E}_p}{\hat{E}_e} = \frac{C_p + C_q q_p}{\hat{C}_e} \tag{34}$$

The numerator of (34) is equal to A and, following (33), is negative. The denominator is the partial derivative of the second period ('future') oil consumption with respect to the second period oil price. In fact, it denotes the same effect as the numerator (it is a derivative of a demand function with respect to the own price) and, therefore, is negative. Hence:

$$e_p < 0 \tag{35}$$

(35) has an appealing intuitive explanation. A higher current oil price implies lower current oil consumption. For given oil reserves $\bar{R}$, it implies a lower future excess demand and, therefore, a lower future oil price.

(35) is only a sufficient condition for G > 0. In general, G will be positive if:

$$\tilde{S}_p > \tilde{S}_e(-e_p) \tag{36}$$

Suppose $\tilde{S}_p = -\tilde{S}_e = b > 0$ where b is the speed of adjustment in the asset adjustment equation (22). In this case, (36) is equivalent to:

$$e_p < 1 \tag{37}$$

which might be interpreted as an assumption postulating Hicksian inelastic expectations. In fact if one considers "small" deviations from the temporary equilibrium where p*=e*, (37) is equivalent to:

$$\frac{e_p p^*}{e^*} = \frac{\partial e/e^*}{\partial p/p^*} < 1 \tag{38}$$

From now on we will assume that either expectations are rational (so that condition (35) holds) or that they are such as to satisfy (35). Both these assumptions imply $G > 0$. Taking (33) into account we can finally write:

$$\tilde{E}_p = A - G < 0 \tag{39}$$

We now turn to $\tilde{E}_m$. From the definition, $\tilde{E} = C(p,q) - \tilde{S}(p,e)$, we have:

$$\tilde{E}_{\bar{m}} = C_q q_{\bar{m}} \tag{40}$$

or, using (15),

$$\tilde{E}_{\bar{m}} = -\frac{H_{\bar{m}}}{H_q} C_q = \frac{d^1_{\bar{m}'} - d^2_{\bar{m}}}{H_q} C_q \tag{41}$$

where $\bar{m}'$ is defined as $\bar{M} - \bar{m}$ and $d^1_{\bar{m}'}$, $d^2_{\bar{m}}$ are the wealth effects on OECD's and OPEC's consumption respectively. Since $C_q > 0$ and $H_q < 0$, $d^1_{\bar{m}'} < d^2_{\bar{m}}$ ($d^1_{\bar{m}'} > d^2_{\bar{m}}$) implies $\tilde{E}_{\bar{m}} > 0$ ($\tilde{E}_{\bar{m}} < 0$) and $d^1_{\bar{m}'} = d^2_{\bar{m}}$ implies $\tilde{E}_m = 0$. As we will see later, the results depend on which of those three cases hold. At the moment we assume $d^1_{\bar{m}'} = d^2_{\bar{m}}$ and accept that $\tilde{E}_{\bar{m}} = 0$.

Next, we will consider the partial derivatives of B' and B'' with respect to p and m. From (25) and (13) we obtain:

$$B'_p = T_p(1 - qd^2_T) - q_p(qd^2_q + d^2) \tag{42}$$

Since we are in the regime of excess demand for oil, $T_p = \tilde{S} + \tilde{S}_p > 0$. $q_p = -H_p/H_q$ (see equation 15) and it is straighforward to show that $q_p > 0$ follows from (32). In fact $q_p > 0$ is what common sense would suggest. A higher oil price is expected to be followed by higher consumption good prices. It remains to explain the terms in the parentheses. For this purpose it will be useful to denote OPEC's expenditure (qd^2) by D. Doing this, we can write (42) as:

$$B'_p = T_p \ (1-D_T) - q_p D_q \tag{43}$$

D_T is OPEC's marginal propensity to spend. Hence $(1-D_T)$ is OPEC's marginal propensity to save and is, therefore, non-negative. The term D_q is the change in expenditure following an increase in the price. Its sign depends on the price elasticity of OPEC's demand (imports) for good d. We will assume that elasticity exceeds unity and accept that $D_q < 0$. As we will explain later, the results do not depend on this assumption.

It follows from the above arguments that:

$$B'_p > 0 \tag{44}$$

Next, we consider the partial derivative B''_p, ie the change in OPEC's trade surplus following an increase in the oil price in the regime of excess oil supply. From (25') and (13) we obtain:

$$B''_p = \overline{T}_p \ (1-D_{\overline{T}}) - q_p \ D_q \tag{45}$$

where $\overline{T} = pC(p,q)$. If we adopt the assumption that OECD's demand for oil C(p,q) is price elastic, $\overline{T}_p < 0$. Having previously assumed that OPEC's demand for good d is also price elastic (and $D_q < 0$) implies that the sign of B''_p is ambiguous. However, we will later see that the results are not sensitive to the sign of B''_p. As a working hypothesis we will assume that in the regime of excess oil supply an increase in the oil price reduces OPEC's trade surplus:

$$B''_p < 0 \tag{46}$$

It remains to find the derivatives $B'_{\overline{m}}$, $B''_{\overline{m}}$. According to the monetary approach to the balance-of-payments these are expected to be negative. From (25) we have:

$$B'_{\overline{m}} = -(q_{\overline{m}}d^2 + d^2_{\overline{m}}\, q) \tag{47}$$

Since we have assumed that wealth effects are the same ($d^1_{\overline{m}} = d^2_{\overline{m}}$), (15) implies $q_{\overline{m}} = 0$ Hence:

$$B'_{\overline{m}} = -\, qd^2_{\overline{m}} < 0 \tag{48}$$

From (25′) we obtain:

$$B''_{\overline{m}} = pC_q q_{\overline{m}} - (q_{\overline{m}}d^2 + d^2_{\overline{m}}\, q) \tag{49}$$

and since $q_{\overline{m}} = 0$,

$$B''_{\overline{m}} = -\, qd^2_{\overline{m}} < 0 \tag{50}$$

THE STABILITY OF THE TEMPORARY EQUILIBRIUM

Here we reproduce the system of differential equations that governs the motion of the economy. This is:

$$\dot{p} = a\tilde{E}, \tag{51}$$

$$\dot{\bar{m}} = \begin{cases} B' \text{ if } \tilde{E} > 0 \\ B'' \text{ if } \tilde{E} < 0 \end{cases} \tag{52}$$

We will study the stability properties of the above system by constructing its phase diagram in the $(p,\bar{m})$ space. The slope of the $\dot{p} = 0$ locus is $-\tilde{E}_{\bar{m}}/\tilde{E}_p$. Since we have assumed that $d^1_{\bar{m}'} = d^2_{\bar{m}}$ it follows from equation (41) that $\tilde{E}_{\bar{m}} = 0$. The $\dot{p}=0$ locus will be a line parallel to the $\bar{m}$-axis starting from the point $(p^*,0)$ where p^* is the price which clears the oil market. Since $\tilde{E}_p < 0$, $p > p^*$ $(p<p^*)$ implies $\dot{p} < 0$ $(\dot{p} > 0)$.

From (52) it is obvious that the $\dot{\bar{m}}=0$ locus has a 'kink' at the point where $\tilde{E}=0$. When excess oil demand $(\tilde{E}>0)$ prevails its slope is positive:

$$\frac{dp}{d\bar{m}} = -\frac{B'_{\bar{m}}}{B'_p} > 0 \tag{53}$$

while in the regime of excess oil supply $(\tilde{E}<0)$ its slope is negative:

$$\frac{dp}{d\bar{m}} = -\frac{B''_{\bar{m}}}{B''_p} < 0 \tag{54}$$

(53) and (54) follow from (44), (46), (47) and (49). Hence, the phase diagram of the system of differential equations (51), (52) will be as in Figure 1. It follows that the temporary equilibrium is stable.

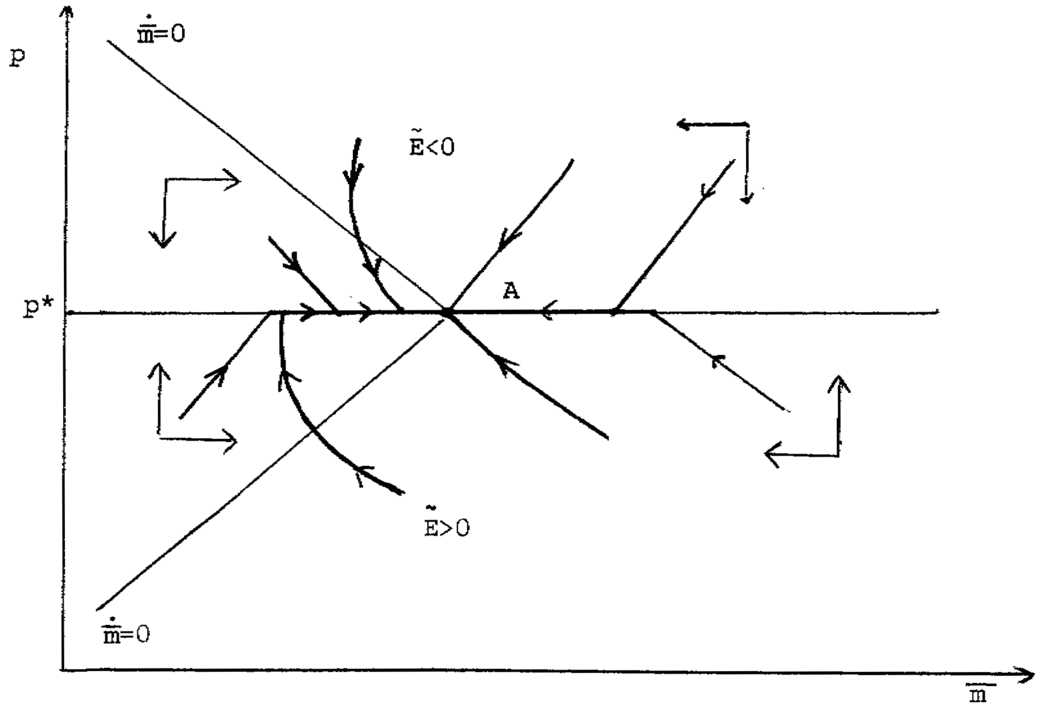


Figure 1

This result is not surprising. System (51), (52) consists of two pairs of differential equations. The pair $\dot{p}=a\tilde{E}$, $\dot{\bar{m}}=B'$ in the regime of excess oil demand ($\tilde{E}>0$) and the pair $\dot{p}=a\tilde{E}$, $\dot{\bar{m}}=B''$ in the regime of excess oil supply ($\tilde{E}<0$). Consider the pair $\dot{p}=a\tilde{E}$, $\dot{\bar{m}}=B'$. Its Jacobian J satisfies the Routh-Hurwitz conditions for stability:

$$\begin{aligned} &\text{trace} && J = \tilde{E}_p + B'_{\bar{m}} < 0 \\ &\text{determinant} && J = B'_{\bar{m}}\tilde{E}_p > 0 \end{aligned} \qquad (55)$$

Similarly, consider the pair $\dot{p}=a\tilde{E}$, $\dot{\bar{m}}=B''$. Its Jacobian $\bar{J}$ also satisfies the Routh-Hurwitz conditions:

$$\begin{aligned} &\text{trace} && \bar{J} = \tilde{E}_p + B''_{\bar{m}} < 0 \\ &\text{determinant} && \bar{J} = B''_{\bar{m}}\tilde{E}_p > 0 \end{aligned} \qquad (56)$$

Therefore, system (51), (52) is characterized by <u>regime stability</u>. Although regime stability is not generally sufficient for the stability of the equilibrium,[10] it is clear from the above diagram (Figure 1) that, in this particular case, regime stability is sufficient for this. As soon as a trajectory crosses the line p=p* it continues on it until it converges to the equilibrium point A (See also Footnote 12).

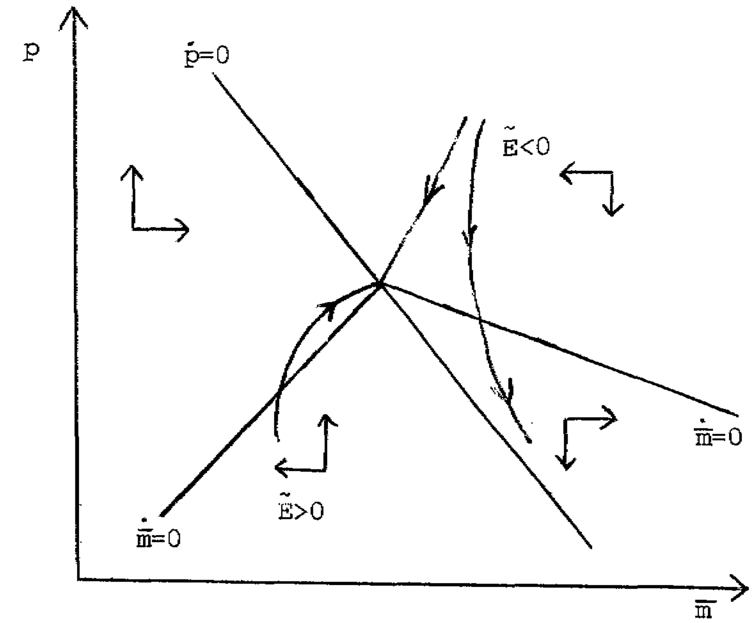
It might be noticed from inspecting (55) and (56) that the above result does not depend on the particular assumptions imposed to derive conditions (44) and (46). That is, the equilibrium is still stable even if $B'_p < 0$ and/or $B''_p > 0$. For instance, the assumptions that oil demand and OPEC's demand for the consumption good are price elastic are not necessary for stability. However, the result does depend on the assumption $d\frac{1}{m'} = d\frac{2}{m}$ ie that an increase in wealth has the same effect on OECD's and OPEC's consumption. For suppose that $d\frac{1}{m'} > d\frac{2}{m}$.[11] From (41), $q_{\bar{m}} < 0$ and, therefore, $\tilde{E}_{\bar{m}} < 0$. As a consequence the $\dot{p}=0$ locus will now have a negative slope. It is easy to verify that in the regime of excess demand ($\tilde{E}>0$) the stationary point of the system of equations $\dot{p}=a\tilde{E}$, $\dot{\bar{m}}=B'$ is stable. However, in the regime of excess oil supply ($\tilde{E}<0$) stability depends on the position of the $\dot{p}=0$ locus relative to that of the $\dot{\bar{m}}=0$ locus. If:

$$\frac{\tilde{E}_{\bar{m}}}{\tilde{E}_p} > \frac{B''_{\bar{m}}}{B''_p} \tag{57}$$

that is, if the $\dot{p}=0$ locus is steeper than the $\dot{\bar{m}}=0$ locus, it is easy to show that the stationary point of the system $\dot{p}=a\tilde{E}$, $\dot{\bar{m}}=B''$

is unstable. The phase diagram for (51), (52) will be as in Figure 2.

Figure 2

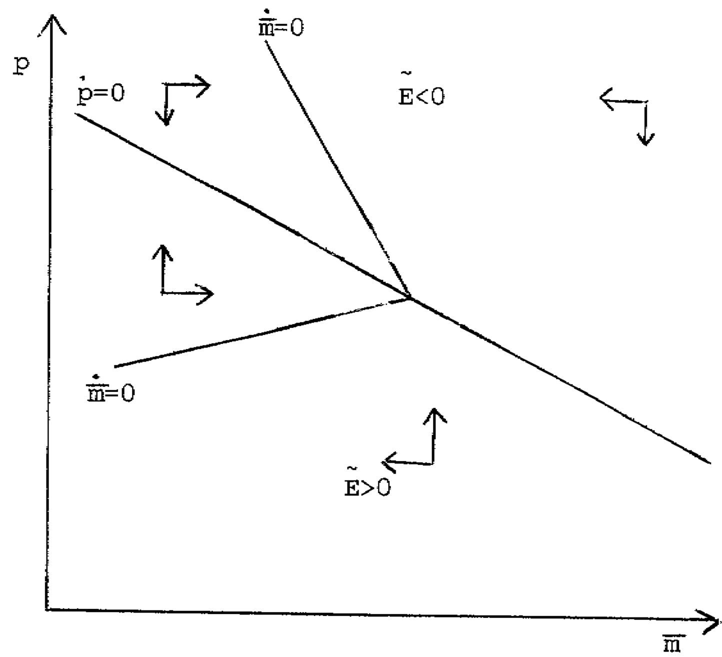


If the $\dot{\bar{m}}$ locus is steeper than the $\dot{p}=0$ locus, that is if:

$$\frac{\tilde{E}_{\bar{m}}}{\tilde{E}_p} < \frac{B''_{\bar{m}}}{B''_p} \qquad (58)$$

it is easy to show that regime stability prevails for both the $\tilde{E}>0$ and the $\tilde{E}<0$ regimes. However, we cannot conclude that the equilibrium is stable by just looking at the phase diagram (Figure 3) since the 'kink' that occurs at $\tilde{E}=0$ might force trajectories to move back and forth between regimes without converging to equilibrium.[12] However it has been shown (Van den Heuvel, op cit, theorem 2.32) that in a piece-wise linear differential system with two variables, regime stability is a sufficient condition for the stability of equilibrium. Hence if $d^{1}_{\bar{m}} > d^{2}_{\bar{m}}$ the equilibrium will still be stable provided that condition (58) holds.

Figure 3



Before we close this section we should make some remarks about the role of expectations in this model. In particular, one might ask whether the temporary equilibrium has any efficiency properties. Although we have assumed that the economy is competitive we cannot conclude that the equilibrium is efficient. Since only spot markets are open in the first period, a sequence of temporary equilibria like A (see Figure 1) might involve a very rapid or a very slow oil extraction. The problem is well known from the theory of exhaustible resources; in the absence of a complete set of forward markets, the short-run equilibrium may be a "myopic" one. (See Dasgupta and Heal, 1979). It is only in the case of rational expectations (ie expectations based on future scarcity) that the temporary equilibrium is efficient. In our two-period economy, expectations are rational if e solves equation (21).

CONCLUDING COMMENTS

In this paper we have developed a two country, non-tâtonnement model of monetary and price adjustment resembling the process of recycling oil revenues. Given the simplicity of the model and some rather plausible assumptions on the formation of expectations we have shown that the equilibrium is stable if the wealth effect on the two country's consumption is the same.

The model can be extended in many directions. For instance, one might wish to study the adjustment process under floating exchange rates; to introduce more than one financial asset (bonds) and non-traded goods (eg labour) and consider also the possibility of rationing in these markets (credit rationing, unemployment, etc); or to consider more than two countries and study the trade relations among them. The introduction of such issues would make the analysis much more interesting and provide a framework where macroeconomic issues could be properly discussed.

However, stability results are difficult to derive under a non-tatonnement adjustment process and in the presence of many non-clearing markets, due to spill-over effects. In this case one should rather concentrate on regime stability and comparative static results within particular regimes. Such results are already available in the literature (see, among others, Marion and Svensson, 1984)

FOOTNOTES

1. f represents expectations about the future price of good d.

2. It is assumed that no oil is stored in OECD.

3. In addition to (6), (7), (8) the solution to the above problem gives a demand for money function. As we will see later, we will not need it because of Walras´ law.

4. Since money earns a zero rate of return, no discounting factor appears in (11). In effect, (12) resembles the Hotelling principle: if there were a credit instrument in the economy earning a rate of return r, (12) would be p=e/1+r, which is the Hotelling principle in discrete time.

5. (12) is the profit maximising condition for an asset management problem and assumes that OPEC acts as a perfect competitor. The solution is not indeterminate since, at equilibrium, S* will be determined by demand (see the equilibrium condition (16) below). If OPEC were modelled as a monopolist, it would take into account oil demand and determine current supply by equating the present to the expected marginal revenue.

6. $\hat{C}(.)$ is an estimate of the future oil demand function. Since oil demand is stationary, C(.) and $\hat{C}(.)$ must have the same functional form.

7. The use of time derivatives and the notion of asymptotic stability are not inconsistent with the Hicksian, two period framework. See Arrow and Hahn (1971) ch.12.9.

8. See, among others, Van den Heuvel (1983), Stournaras (1982).

9. This seems a reasonable assumption given the experience following the oil price shocks of the seventies.

10. See references in Footnote 8.

11. The case $d^1_{\overline{m}'} < d^2_{\overline{m}}$ can be analyzed along similar lines.

12. The problem is familiar from the literature on regime switching (see Van den Heuvel, op cit, for a survey and generalization of such results). It might be noticed that in the case where $d^1_{\overline{m}}$, = $d^2_{\underline{m}}$ (shown in Figure 1) we did not encounter such a problem since no trajectory could ´escape´ from the line p=p* once entered into it. In other words the line p=p* is a ´strongly attractive´ set and, therefore, the equilibrium is stable (Eckalbar, 1980).

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