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Capacity Constraints and the Production of Nonrenewable Resources

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ABSTRACT

This paper examines the extent to which the use of a fixed input in the extraction process for a nonrenewable resource affects a number of common results in depletion theory. A multi-deposit model is constructed where the extraction technology requires capital equipment that is deposit-specific and has no resale value once installed. It is shown that for each deposit there is an equilibrium capacity level, which is built up all at once or gradually, depending on the adjustment costs associated with installing the equipment and the heterogeneity of deposits. The aggregate extraction rate is constant over an initial period of time.

The paper goes on to derive a number of results for this model. Firstly, different quality deposits are always exploited simultaneously, although better quality deposits are exhausted first. Secondly, higher discount rates entail quicker depletion only if the resource is sufficiently scarce relative to capital equipment. Thirdly, a programme that is optimal from a social viewpoint can in principle be reproduced without intervention in a perfectly competitive market. Fourthly, fiscal instruments generally discourage investment, and consequently overconserve the resource, unless tax writeoff provisions or depletion allowances are in force. Fifthly, exploitation under monopoly and symmetrically placed Cournot-Nash producers is generally more conservationist than the social optimum, but the distortion is negligible if the number of producers is sufficiently large. Finally, allowing for a variable input in the extraction process or a positive rate of depreciation for capital equipment is shown to qualify the results by restoring the more common result that the aggregate extraction rate is always a strictly declining function of time.

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1. INTRODUCTION

This paper is concerned with the implications of the use of fixed capital in the extraction process for an exhaustible resource. Much, though not all, of the theoretical literature on resource depletion ignores these implications by assuming either (i) costless extraction; or (ii) a cost function that is increasing and convex in the rate of extraction, with the property that it costs nothing to extract nothing.

The second of these two assumptions is equivalent to the supposition that all factors of production used in the extraction process are costlessly and instantaneously variable (henceforth just "variable" or "malleable"), and that the process exhibits diminishing returns.[1] In terms of the results generated, models incorporating this assumption do not add much beyond models of costless extraction (the extraction rate still falls over time, but marginal profit rather than price grows at the rate of r per cent), except insofar as they highlight the presence of the interest rate in the cost function. The interest rate enters as the opportunity cost of capital services employed in the extraction process. As a result, a rise in the interest rate does not, as originally argued, have a clear-cut effect on the pace of depletion. The increased gain associated with extracting now rather than later is offset by the increased cost occasioned by the additional capital services needed to extract now. Which effect outweighs the other depends, among other things, on the size of remaining reserves. This point is discussed by Lasserre

(1985a), and especially by Farzin (1984) in the case of a constant returns extraction technology. Farzin also demonstrates that, if there is a conventionally produced substitute for the resource, it may be that a larger interest rate is <u>always</u> associated with greater conservation, contrary to the previously accepted result.

If, however, capital is non-malleable, a closer look at decisions about factor service purchases is in order before anything is posulated about the shape of the extraction cost function. In this paper, it will be assumed (except in section 5A) that that there are no adjustment costs associated with installing capital equipment. Instead the term "non-malleable" will be taken to mean "never worth removing once installed". This assumption - preserved throughout this paper - is justified in the absence of workable second-hand markets for the types of capital equipment, often deposit-specific, that are used in the extraction process. In this case, the equipment has no resale value. Alternatively, some of the equipment may be prohibitively costly to remove from the site once installed. A related possibility is that equipment can only be removed in one discrete block once exploitation of the resource deposit has definitively ceased.

A few authors have studied the implications of the use of non-malleable capital equipment in the extraction process. Unless otherwise specified, all studies cited here pertain to the exploitation of a single deposit by a single firm. In Campbell (1980) the firm faces a time-invariant net revenue function that is strictly concave in the rate of extraction, and capacity installation is irreversible. He shows that, in the absence of adjustment costs, the firm picks its optimal capacity at the outset and installs the required capital equipment instantaneously. It then proceeds to exploit the deposit at full capacity for an initial period of time. Over a subsequent and final period, the firm sustains a time-decreasing rate of extraction until its deposit is exhausted. However, Crabbé (1982) uses the same construct to demonstrate that if the assumption of a strictly concave net revenue function is dropped, the firm exploits its deposit at maximum capacity up to exhaustion.

Lasserre (1985a) considers a firm with a multi-factor diminishing returns extraction technology. He begins by showing that, if all factors are variable (whether denominated in stock or flow dimensions), the usual result that the extraction rate falls over time remains valid. This is hardly surprising: the cost function dual to the technology is convex in the output rate, so for given input prices the extraction problem is just the usual one. More interesting is the case where additions to the stock of capital equipment are irreversible, or involve (separable) adjustment costs. In the former case, the time-decreasing extraction result stays. Again, it is straightforward to see why: for a given stock of the fixed factor(s), the restricted cost function that is dual to the technology is strictly convex in the extraction rate. In the latter case, the equilibrium extraction profile may be everywhere time-decreasing (if the inherited capital stock is "large"), or single-peaked (if the inherited capital stock is "small" and has to be built up initially). Finally, Lasserre shows that if initial capacity is a choice variable at the initial date but subsequent reductions in it incur adjustment costs, the time-decreasing extraction result resurfaces.

The problem of extraction where changes in capacity are subject to adjustment costs has also been formulated by Gaudet (1983). For a firm facing a constant resource price, he shows that in general capacity is built up smoothly over an initial phase, peaks, and is thereafter run down gradually. So long as the extraction rate is positive, it is equal to capacity output, but there may be a final phase of dismantlement when the firm's reserves are exhausted.

A number of the foregoing results have been derived in an earlier paper by Jacobson and Sweeney (1980). In their construct, a group of price-taking firms accumulate a "capital" stock in the form of a cumulative stock of wells drilled (say W). Any increase in W reduces the cost of extracting a given amount. This interpretation of the "capital stock" helps to rationalize the convex adjustment cost function associated with increases in W.[2] Extraction costs are assumed to be linearly homogeneous in the rate of extraction (say R) and W.

The paper shows that R/W increases (declines) whenever price grows at a faster (slower) rate than the interest on the marginal profit; however, R may increase even if price growth is zero or negative. In general, W is increased on an initial phase, and R rises alongside. R typically declines on a later phase, unless price growth is very rapid. The paper also derives a number of comparative dynamic results. Chief among these is that a larger interest rate has an ambiguous effect on the initial rate of extraction, and that a larger reserve base lengthens the extraction horizon and raises the equilibrium R and R/W at each moment in time. The paper's weakness, however, is that it takes the resource price and its growth rate to be exogenously determined.

This paper adds to the work reviewed above by introducing explicitly several distinct resource deposits, which may differ in quality (accessibilty). To examine the case of several different deposits seems important because, particularly where capital is used in the extraction process, the conditions under which capital and resource stocks can be aggregated for the purposes of analysis are very stringent (Blackorby and Schworm, 1980).[3] In any case, an analysis of the aggregate output profile masks the time-allocation of this output across deposits.

In addition, the analysis here endogenizes the resource price, and thereby circumscribes its possible time-path. The paper also contains a number of comparative dynamic results not previously derived for the case where non-malleable capital is used in the extraction process. These include results about the interest rate effect, tax instruments, and non-competitive exploitation of resource deposits.

The plan of this paper is as follows: in the next section (2) the resource deposits are assumed to be publicly managed. Subsection 2A poses the general problem of optimal capacity choice and outlines the main features of the solution. Subsection 2B turns to a two-deposit example to examine these features in greater detail. Subsection 2C uses the simplifying assumption of identical deposits to derive some comparative dynamic results with respect to changes in the parameters of the model.

Section 3 verifies that, under appropriate conditions, competitive exploitation of the resource deposits duplicates the outcome under social management. Section 4 turns to the issue of deviations from the optimal programme, again using the assumption of identical deposits. Subsection 4A investigates the effects of commonly used taxes and allowances on the competitive outcome. Subsections 4B and 4C compare the outcome when the deposits are exploited by a monopolist and Cournot-Nash oligopolists, respectively, with the outcome under competition.

Section 5 returns to the public management framework to investigate the effects of dispensing with three of the simplifying assumptions in section 2. Subsection 5A introduces adjustment costs into the capacity expansion problem, subsection 5B removes the assumption that no malleable factors are used in the extraction process, and subsection 5C introduces depreciation. Finally, section 6 contains a few concluding remarks.

2. CAPACITY CHOICE AND DEPLETION IN THE SOCIALLY MANAGED INDUSTRY

A. Formulation and Solution of the General Problem

This section investigates the equilibrium extraction profile of a publicly managed exhaustible resource. The objective is to maximize the discounted stream of consumer plus producer surplus. The (homogeneous) reserves of the resource are assumed to be distributed among N separate resource deposits. Deposit i $(i=1,..,N)$ holds a stock of known and fixed size, S_{i0}, at the initial date. Production from i at date $t>0$ is subject to the capacity constraint

$$R_i(t) \leq f_i(K_i(t))$$

where R_i denotes output from deposit (occasionally "field") i and K_i denotes the volume of the single non-transferable input - capital equipment - in place there.[4] The following is assumed:

(A.1) $f_i(K_i)$ is continuous with $f_i(0)= 0$ and continuous first and second derivatives satisfying $f_i'(K_i)>0$ and $f_i''(K_i)<0$ for $K_i \geq 0$, $i = 1,...,N$.

Now define $u(R) = \int_0^R p(z)dz$, where $p(.)$ denotes the inverse demand function for the resource. Its assumed properties are:

(A.2) $p(R)$ is continuously differentiable, unchanging over time, and satisfies $\lim_{R \to 0} p(R)=+\infty$.

where $R = \Sigma_i R_i$ is the total resource flow. Let q denote the known and constant market price per unit of capital equipment. The supply of capital is assumed perfectly elastic at this price. Also

(A.3) Capital equipment is infinitely lived.

(this is relaxed in section 5C below) and

(A.4) The only cost associated with installing a unit of capital equipment is the purchase price (q) for that unit. Once installed, however, a unit has no resale value.[5]

Suppose for convenience that there is no capital equipment in place at any of the deposits at the start of the planning horizon. The public agency's criterion is then to choose a profile $\{R_i(t), I_i(t)\}^{\infty}_{t=0}$ to maximize

$$(1) \quad \int_0^{\infty} e^{-rt} \{u(\Sigma_i R_i(t)) - q\, \Sigma_i I_i(t)\}\, dt,$$

subject to

$$(2) \quad \dot{S}_i(t) = -R_i(t) \quad S_i(0) = S_{i0} \text{ given}, \lim_{t \to T_i} S_i(t) \geq 0;$$

$$(3) \quad \dot{K}_i(t) = I_i(t),\ K_i(0) = 0;$$

$$(4) \quad 0 \leq R_i(t) \leq f_i(K_i(t)); \quad \text{and}$$

$$(5) \quad I_i(t) \geq 0$$

for $i = 1,..,N$, where I_i denotes investment in capital equipment at field i, and r denotes the social rate of discount (the "interest rate"), assumed time-invariant. Constraint (2) captures nonreplenishability, (3) simply defines investment (using (A.3)), and (5) rules out disinvestment.

Using standard methods, there exist continuous functions μ_i and λ_i such that a solution satisfies, for $t \geq 0$,

(6) $e^{-rt}p(R) - \mu_i \geq 0, \quad R_i \leq f_i(K_i)$ (CS) and

$e^{-rt}p(R) - \mu_i < 0$ implies $R_i = 0$;

(7) $e^{-rt}q \geq \lambda_i, \quad I_i \geq 0$ (CS); and

(8) $\dot{\lambda}_i = - \{e^{-rt}p(R) - \mu_i\} f_i'(K_i)$

(where time-arguments have been omitted for convenience) as well as (2), (3), μ_i constant, and $\lim_{t \to T_i} \lambda_i(t) = 0$ for $i = 1, \ldots, N$. The letters (CS) indicate that the inequalities hold with complementary slackness, and $0 < T_i \leq \infty$ is the date at which deposit i definitively ceases production. Condition (6) states that the deposit should be exploited at maximum capacity if the (PV) resource price is larger than the (PV) marginal value of a unit of stock <u>in situ</u> (this discounted "user cost" is constant over time because the own rate of return on the resource is zero). Conversely, nothing should be produced if the inequality is reversed. If the two are equal, the rate of extraction from deposit i is indeterminate. Conditions (7) and (8) assert that if the investment rate for deposit i is positive, it must equate the discounted "marginal value product" stream of returns to an incremental unit of equipment with the incremental purchase cost. If the purchase cost is larger, the investment rate is zero. Because the Lagrangian function corresponding to (1)-(5) has been assumed concave in the vector of control and state variables, the set of conditions (2)-(8) with the boundary conditions on the λ_i's is also a set of sufficient conditions for a welfare-maximizing extraction programme (Long and Vousden, 1977, Theorem 7).

What can be said about the solution at this level of generality? A few characteristics can be deduced under (A.1) - (A.4). Firstly, assumption (A.2) ensures that the resource will not be exhausted in finite time. That is, at least one of the T_i is infinite. Secondly, Appendix A, part (i) demonstrates that a positive (but finite) investment rate for any deposit can only be observed if the rate of growth of the resource price is positive. In the special case of N identical deposits, this does in fact rule out intervals of positive investment altogether (that is, any expansion must involve adding a block of capital equipment all at once; see parts (ii) and (iii) of the Appendix). In the case of dissimilar deposits, the assumption that inherited capacity in every deposit is sufficiently small can be used to rule out positive (finite) investment in any deposit on an initial phase. Thirdly, any increase in K_i, i=1,..,N, in the form of a discrete jump - at a jump, the rate of investment is momentarily infinite - can only occur at the initial date t=0. This is due to the strict concavity of u(.) and $f_i(.)$, i=1,..,N, and is shown in part (iii) of Appendix A.

As regards the behaviour of the resource price, the optimal programme is characterized by two types of phases (see the system of complementary equations and inequalities (6)). Consider first a phase on which $\dot{p}\leq 0$. Capacity expansion cannot be observed on such a phase (recall that $\dot{p}>0$ is a necessary condition for a positive investment rate on any deposit). Together with assumption (A.3), this implies that capacity in every deposit remains the same on this phase. Further, because $\dot{p}\leq 0$, it pays to operate every (unexhausted) deposit at capacity. Resource production is thus constant on this phase, so $\dot{p}=0$ must be observed.

Consider next a phase on which $\dot{p}>0$. Since total output is declining, the rate of extraction from at least one deposit (say j) is positive and declining at any time during this phase. But, from (6), this implies $e^{-rt}p(R)=\mu_j$, and thus r per cent price growth. Thus a phase on which price growth is positive must automatically feature the Hotelling rule.

Next, an optimal programme cannot feature a discontinuity in the resource price at any date. If contrary to this there is a discontinuous increase in p at some date T^d, the value of the programme could be increased by relocating a unit of extraction from just before T^d to just after it. Clearly this relocation is feasible: since output falls discontinuously at T^d, some capacity must lie idle immediately after this date. Similarly, a discontinuous fall in p at date T^d (a discontinuous increase in output) means that at least one deposit was operating at less than full capacity just prior to T^d (recall that no "bang-bang" expansion is allowed after the initial date). But then an increase in the objective function is achieved by increasing output slightly just prior to T^d, which is a feasible perturbation. These arguments hold so long as there is any discontinuity in p at any date, so a discontinuity cannot characterize equilibrium, as asserted.

Finally, a phase on which all N deposits are worked at maximum capacity - which implies a constant price (marginal value) for the resource - cannot be preceded by a phase on which (at least) one deposit is operated at less than capacity. Assume that it can be, and suppose that deposit j is exploited at less than capacity for an interval of time prior to τ, and at capacity for a period of time after τ, where τ is the transition date from one phase to the other. Then, for some time after τ, the marginal value of the resource is constant, and so its PV must be declining. It therefore pays to reallocate extraction to some date just before τ, when the PV of marginal value is larger, so long as there is spare capacity at any deposit then. The assertion at the beginning of the paragraph thus holds.

The following subsection analyzes the properties of equilibrium depletion profiles in greater detail by referring to a two-deposit example. In particular, it points out that the optimal programme will always feature an interval of time on which different "quality" deposits are exploited simultaneously.

B. A Two-Deposit Example

Suppose N=2, and again let initial capacity in each of the two deposits be "sufficiently small". Since by assumption there is no chokeoff price, both deposits will be exploited and exhausted asymptotically. The argument in the previous section tells us that a block of capital equipment will be installed in at least one of the two deposits at the initial date. Furthermore, the optimal programme will feature an initial phase (say phase 1) on which the resource price is constant and both deposits are operating at capacity (perhaps zero for one of the deposits).

Since reserves are finite, phase 1 is eventually followed by a phase (say phase 2) on which price grows at the rate of r per cent. At every point during phase 2, output from at least one deposit must be declining. Suppose that output from deposit 1 is declining over (at least) the initial portion of phase 2. It is then immediate that no further capacity expansion can ever take place in deposit 1. This follows from the fact that $e^{-rt}p=\mu_1$ over a period of time. Since price can never grow by more than r per cent, $e^{-rt}p\leq\mu_1$ at all subsequent dates. Thus $\lambda_1=0$ from the beginning of phase 2 onwards, and additional capital equipment in deposit 1 is worthless.

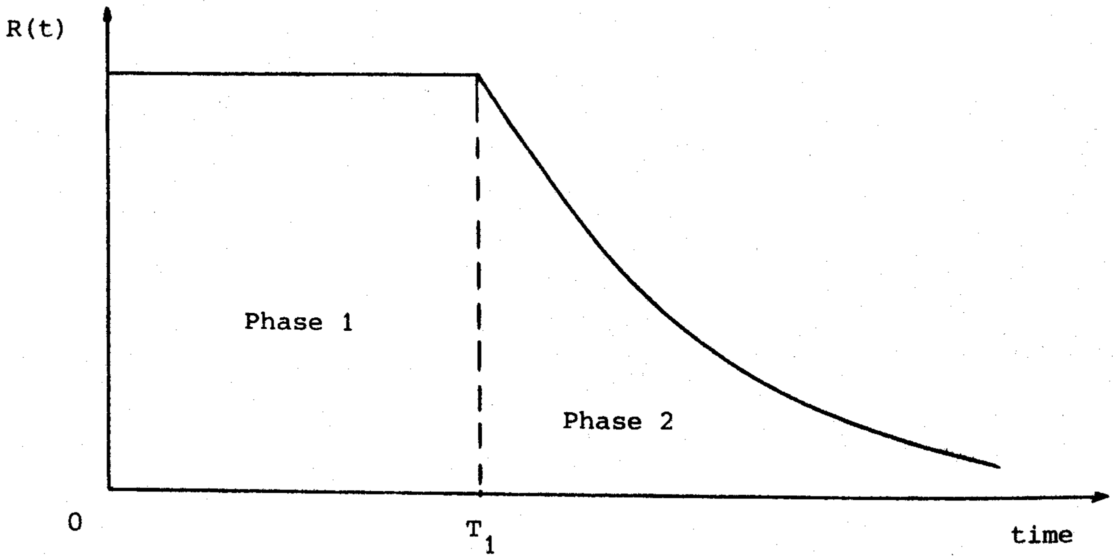
Phase 2 may, however, feature capacity expansion in deposit 2. If it does, two conditions must be satisfied:

(i) Output from deposit 1 must be declining while the capital stock in deposit 2 is expanding. This implies that $\mu_2<\mu_1$; that is, an incremental unit of stock *in situ* at the inital date is more highly valued if it occurs in deposit 1. Loosely speaking, deposit 1 can then be referred to as the better "quality" deposit.

(ii) Phase 2 - which lasts, say, from T_1 to T_2 - must be followed by a further phase (phase 3, which lasts from T_2 to T_3, say) on which the resource price is constant. For if phase 2 is the final phase, then $e^{-rt}p=\mu_1$ always (recall that price jumps are not permitted), implying that $e^{-rt}p>\mu_2$ always, so that it is optimal to exhaust deposit 2 at full capacity. This implies a discontinuous increase in price at the (finite) exhaustion date, which is a contradiction. In addition, it must be the case that deposit 1 is exhausted at the beginning of phase 3.[6]

Thus if phase 2 features expansion in (at most) one deposit, it cannot be the final phase. In fact, a sufficient condition for the occurrence of phase 3 is that $\mu_2<\mu_1$ (i.e., that the deposits should differ in "quality"), irrespective of whether or not capacity in deposit 2 expands during phase 2. Only in the case $\mu_1=\mu_2$ is phase 2 the final phase. Figures 1 and 2 depict the two possibilities. If phase 3 exists, it must of course be followed by a final phase on which the Hotelling rule holds.

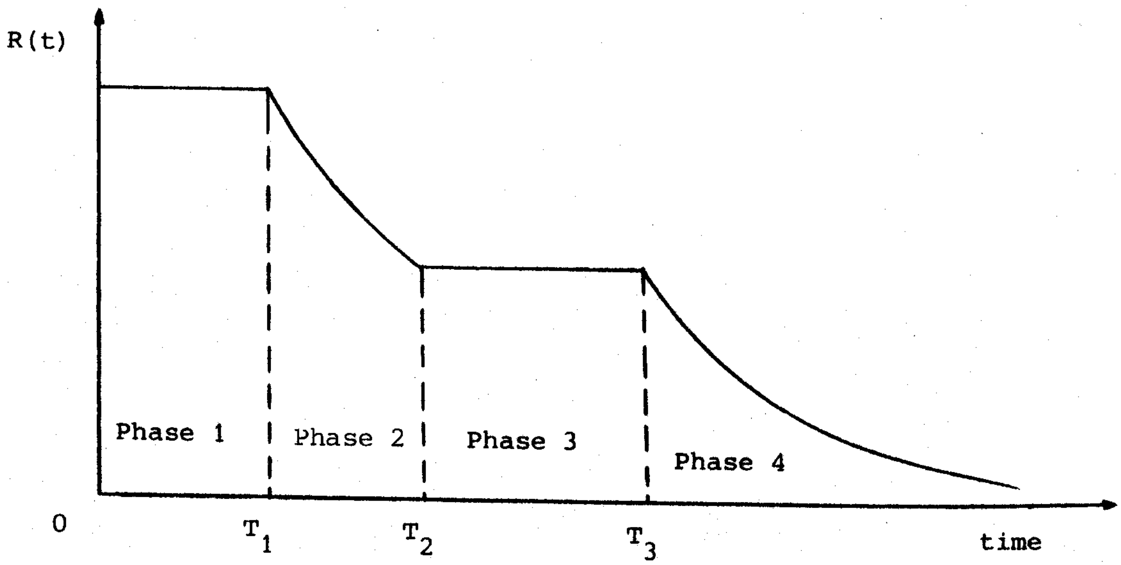
To sum up, even if the deposits differ in quality, the optimal programme always features an interval of simultaneous extraction. For the better quality deposit, all capacity buildup occurs at the initial date. Some, all, or no capacity buildup occurs for the "inferior" deposit. In the first two cases,



Phase 1: Both deposits operate at capacity; capacity is non-zero for both deposits.

Phase 2: Output from at least one deposit declining at any date; Hotelling rule holds.

FIGURE 1 ($\mu_1 = \mu_2$)



Phase 1: Both deposits operate at capacity; capacity may be zero for deposit 2.

Phase 2: Output from deposit 1 declining; Hotelling rule holds. Capacity may expand in deposit 2.

Phase 3: Deposit 2 operates at capacity; deposit 1 exhausted.

Phase 4: Output from deposit 2 declining, Hotelling rule holds.

FIGURE 2 ($\mu_1 > \mu_2$)

simultaneous extraction occurs trivially on an initial interval of time. In the third case, capacity in the inferior deposit expands sometime during phase 2, when output from the better quality deposit is positive but declining. In this case also, then, there is an interval of simultaneous extraction. This result contrasts with the well-known result, where unit extraction cost in each deposit is constant and there are no capacity constraints, that extraction is never simultaneous and that the better deposit is always exhausted first (Herfindahl, 1967, pp. 72-75 and Dasgupta and Heal, 1979, pp. 172-175; see also Hartwick, 1978, for an extension to the N-deposit case, and Solow and Wan, 1976, for an analogous result in an aggregative model with production and consumption where the resource stock is made up of a continuum of grades). Hartwick et al. (1986) establish a similar proposition where a fixed setup cost must be incurred before extraction from a given deposit can begin.

There are, however, some precedents for the simultaneous extraction result (apart from results that rely on imperfectly competitive market structures). Hung (1986) demonstrates that a strictly convex extraction cost function in at least one deposit suffices to rule out strict sequencing: an interval of simultaneous extraction always separates phases of solitary extraction (if any). In a somewhat different context, Ulph (1978) shows that if the resource stock exhibits a continuum of grades, where grade is captured by the magnitude of the (constant) extraction cost, a phase on which different grades are exploited simultaneously may arise. The central assumption is that, for at least part of the stock, a "preparation" cost has to be incurred before a unit of the resource can be extracted. However, grades cannot be segregated beforehand, so the prepared unit appears in a mix of grades, and it pays to extract the proportion of that unit that is above a particular grade - rather than only the minute proportion that is of the very top grade - before "preparing" a further unit. DeMeza and Ungern-Sternberg (1978) demonstrate that simultaneous extraction may arise under future price uncertainty (high-cost owners are unwilling to bear the risk associated with the conservation that is required of them), but the result does not arise under social management or if owners have diversified portfolios (because stock market equity is issued, for example). Finally, Kemp and Long (1980) show in an aggregative model that if the unit cost of extraction is constant in terms of the resource rather than utility, under certain conditions the order in which deposits are exploited is a matter of indifference. However Lewis (1982) shows that, if the resource above ground can be converted into durable capital that provides future consumption as well as into current consumption, the result that deposits should be extracted in order of increasing cost resurfaces.

Finally, it is worth pointing out that although in the present context the two deposits are, over some period of time, exploited simultaneously rather than in strict sequence, the "higher quality" deposit is still unmistakably identified. It is the deposit that is exhausted first. This particular feature is reminiscent of the sequential extraction models. Note also that the simultaneous extraction result here does not depend on initial capacity in the two deposits being zero or "small". For example, if initial capacity in both deposits is positive, then trivially there is an initial period of simultaneous extraction. If, on the other hand, initial capacity in deposit 1 is "large" but initial capacity in deposit 2 is zero, then either some capital equipment is installed in deposit 2 at the initial date, or capacity expansion in deposit 2 takes place gradually (which requires a positive extraction rate from deposit 1), or both. In every case, there is a period during which extraction from both deposits is positive.

C. Comparative Dynamics

To facilitate the analysis, it will be assumed for the remainder of this section that deposits are identical in terms of their physical characteristics and reserves. That is:

(A.5) $f_i(.)=f(.)$ and $S_{i0}=S_0$, $i=1,..,N$.

However, though the qualitative substance of the results that will shortly be derived under the auspices of (A.5) are for the most part generally valid, the shape of the aggregate extraction profile will not in general be as simple as suggested below.

The solution to the problem of maximizing (1) subject to (2) - (5) is particularly simple when assumption (A.5) supplements (A.1) - (A.4). It features instantaneous adjustment to the desired volume of capital equipment $K_i=K^*$ in each and every deposit at the initial date t=0. In contrast to the more common result that the aggregate resource extraction rate is a continuously declining function of time, the programme here consists of two phases. Over an initial interval of time (0,T) (where, of course, T is to be determined), all fields operate at full capacity, and total production of the resource is given by $Nf(K^*)$. The price of the resource is constant at $p(Nf(K^*))$ during this phase. For the subsequent and final interval (T,∞) there is at any date at least one deposit for which capacity is no longer a binding constraint.[7] Industry output falls continuously and price exhibits the familiar Hotelling rule: it appreciates at the percentage rate r. The resource price is continuous at all dates.

We now proceed to characterize K*. The PV of benefits generated by a per-field scale of f(K) can be written as

$$(12)\quad V = \int_0^T e^{-rt}\ u(Nf(K))\ dt + \int_T^{\infty} e^{-rt}\ u(D(p(Nf(K))e^{r(t-T)}))\ dt$$

where D(.) is the inverse function of p(.). Naturally (12) is defined subject to the resource stock constraint

$$(13)\quad NS_0 = TNf(K) + \int_T^{\infty} D(p(Nf(K))e^{r(t-T)})\ dt.$$

(Note that although, for t>T, total output of the resource is given by $D(p(Nf(K))e^{r(t-T)})$, the distribution of this total across fields is indeterminate.[8]) K* is then simply the solution to $\max_K \{V - NqK\}$. It is therefore defined implicitly by

$$(14)\quad V_K = Nq$$

where V_K denotes the derivative of V with respect to K. From (12)

$$(15)\quad V_K = \frac{(1-e^{-rT})}{r}\ p(Nf(K))Nf'(K) + \{p'(Nf(K))Nf'(K)$$

$$-r\frac{dT}{dK}\ p(Nf(K))\}e^{-rT}p(Nf(K))\ \int_T^{\infty} D'(p(Nf(K))e^{r(t-T)})e^{r(t-T)}dt,$$

bearing in mind that T is for given S_0 a function of K, as indicated by (13).[9] Implicit differentiation of equation (13) gives

$$(16)\quad \frac{dT}{dK} = \frac{N\{Tf'(K)+p'(Nf(K))f'(K)\int_T^{\infty} e^{r(t-T)}D'(p(Nf(K))e^{r(t-T)})dt\}}{rp(Nf(K))\int_T^{\infty} e^{r(t-T)}D'(p(Nf(K))e^{r(t-T)})dt} < 0$$

Substituting (16) into (15), a little manipulation establishes that

(17) $V_K = \frac{N}{r}(1-(1+rT)e^{-rT})\ p(Nf(K))\ f'(K)$.

Using (17) in (14), the optimal choice (K^*,T^*) is the simultaneous solution of

(14') $\frac{N(1-(1+rT)e^{-rT})}{r}\ p(Nf(K))f'(K) = Nq$

and (13). For a given extraction technology, demand conditions, and number of deposits, the solution can be denoted $K^*=K(r,S_0,q)$, $T^*=T(r,S_0,q))$. It is, moreover, easy to confirm that $f''(K)<0$ is a sufficient condition for a maximizing solution.

The remainder of this section derives some comparative dynamic results. To begin with, from (17)

(18) $\frac{\partial K}{\partial q}\{V_{KK} + V_{KT}\ \frac{dT}{dK}\} = N$.

Since $V_{KK}<0$, $V_{KT}>0$ (from (17)) and $dT/dK<0$ (from (16)), one infers from (18) that

(19.1) $\frac{\partial}{\partial q}\ K(r,S_0,q) < 0$; and

(19.2) $\frac{\partial}{\partial q}\ T(r,S_0,q) > 0$.

This is, of course, the expected result. A larger acquisition cost for capital equipment reduces installed capacity. For given deposit size, this has the effect of lengthening the period over which the representative deposit is exploited at maximum capacity. This implies a higher resource price over an initial period of time and retards the pace of depletion.

Next, consider the sensitivity of the solution to the size of reserves in the representative deposit. Implicit differentiation of the equilibrium conditions (13) and (14') with respect to S_0 yields that

$$(20)\quad \begin{bmatrix} Nf'(K)\{T+p'(.)\int_T^{\infty} D'(.)e^{r(t-T)}dt\} & -rp(.)\int_T^{\infty} D'(.)e^{r(t-T)}dt \\ \frac{N(1-(1+rT)e^{-rT})}{r}\{Np'(.)f'(K)^2+p(.)f''(K)\} & rTe^{-rT}p(.)f'(K) \end{bmatrix} \times \begin{bmatrix} \partial K/\partial S_0 \\ \partial T/\partial S_0 \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix}$$

where the argument of $D'(.)$ is $p(Nf(K))e^{r(t-T)}$ and the argument of $p(.)$ and $p'(.)$ is $Nf(K)$. The terms are evaluated at the optimal solution (K^*,T^*). From the matrix equation (20) it emerges that

$$(21.1)\quad \frac{\partial}{\partial S_0} K(r,S_0,q) > 0, \text{ and}$$

$$(21.2)\quad \frac{\partial}{\partial S_0} T(r,S_0,q) > 0.$$

That is, installed capacity in the representative deposit is an increasing function of its size; nonetheless this effect is not sufficient to prevent a larger deposit being exploited at maximum capacity over a longer period initially. But larger reserves do - as expected - imply that the resource price $p(Nf(K))$ is lower on an initial interval of time.

The following results are also used below:

$$(22.1)\quad \lim_{S_0\to\infty} T(r,S_0,q) = +\infty; \text{ and}$$

$$(22.2)\quad \lim_{S_0\to 0} T(r,S_0,q) = 0$$

for $r,q>0$. To verify this, suppose to the contrary that T is bounded above by a finite number $\bar{T}>0$ as S_0 becomes arbitrarily large. Fix $T=\bar{T}$ and define $K=\bar{K}$ such that $(\bar{K},\bar{T})$ solves (14') for given r and q. Clearly $\bar{K}$ is finite if $\bar{T}$ is. But then the right-hand side of (13) is finite, so reserves are not fully used up and this contradicts optimality. Similarly, if $\underline{T}>0$ denotes a lower bound that T approaches as reserves are shrunk to zero, define $\underline{K}>0$ such that $(\underline{K},\underline{T})$ solves (14'). But then the right-hand side of (13) is positive, implying, contrary to the initial supposition, that reserves are positive.

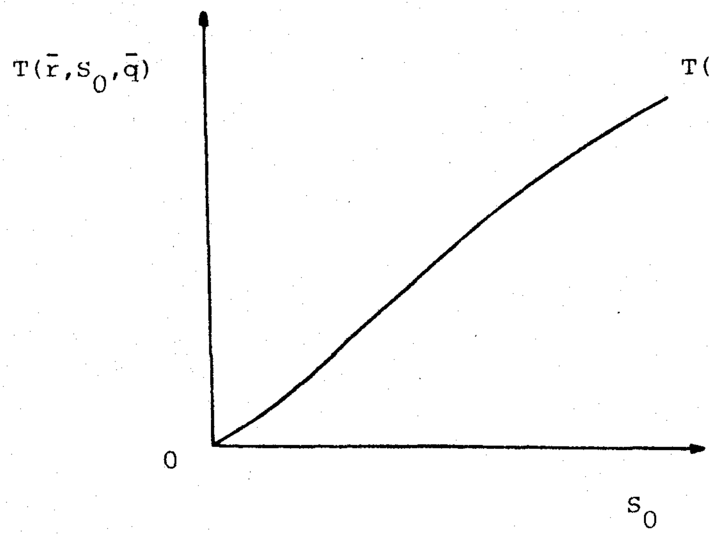
It can be verified similarly that the limit conditions

$$(23.1)\quad \lim_{q\to\infty} T(r,S_0,q) = +\infty; \text{ and}$$

$$(23.2)\quad \lim_{q\to 0} T(r,S_0,q) = 0$$

for $r,S_0>0$, are satisfied. The function $T(r,S_0,q)$ thus has the general shape depicted in Figure 4.

More interesting is the question of the sensitivity of installed capacity (and consequently the rate of depletion) in the representative field to the rate of interest. Propositions 1 and 2 assert that the sign of this depends on the values of S_0 and q. The method of proof is the same for the two propositions (except where indicated), so the two are verified simultaneously. The following definition is used here: for any pair of depletion profiles (say 1 and 2) that begin with the same reserves at the



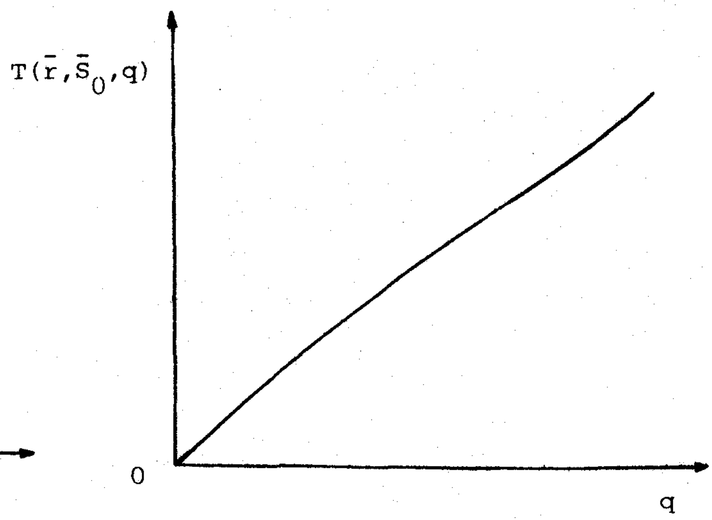


FIGURE 4.1 FIGURE 4.2

initial date, profile 1 is said to be strongly more (less) conservationist than profile 2 if after the initial date, reserves are larger (smaller) at all dates on profile 1 than on profile 2. Profile 1 is said to be weakly more (less) conservationist than profile 2 if, after the initial date, remaining reserves are larger (smaller) for an initial interval of time on profile 1 than on profile 2. Then:

Proposition 1 For given values of r and q, there exists a critical deposit size $So^*>0$ such that $\partial K/\partial r > (<)(=) 0$ if and only if $So < (>)(=) So^*$. For $So<So^*$, a larger interest rate implies a strongly less conservationist depletion policy. Conversely, for $So>So^*$, a larger interest rate results in greater conservation, but only in the weak sense.

Proposition 2 For given values of r and So, there exists a critical value of the purchase price for capital equipment, say

q^*, such that $\partial K/\partial r$ > (=)(<) 0 if and only if $q<$ (=)(>)q^*. For $q<q^*$, a larger interest rate implies a strongly less conservationist depletion policy. However, for $q>q^*$, a larger interest rate results in greater conservation, but only in the weak sense.

Proof From the equilibrium condition (14),

$$\frac{\partial}{\partial r} K(r, S_0, q) = -V_{Kr}\left(V_{KK} + V_{KT}\frac{dT}{dK}\right)^{-1}$$

and since the term in parentheses on the right-hand side is negative,

$$\text{sign}\left(\frac{\partial K}{\partial r}\right) = \text{sign}\,(V_{Kr}).$$

But, using (17),

$$V_{Kr} = -N\left\{\frac{1-(1+rT+(rT)^2)e^{-rT}}{r^2}\right\} p(Nf(K))f'(K)$$

where this derivative is evaluated at (K^*, T^*). While its sign is invariant to the value of K^*, it does depend on the value of T^*, because T^* determines the sign of the term in curly brackets (see Figure 5). For $T^*<T_c$, V_{Kr} is positive, and the converse for $T^*>T_c$.

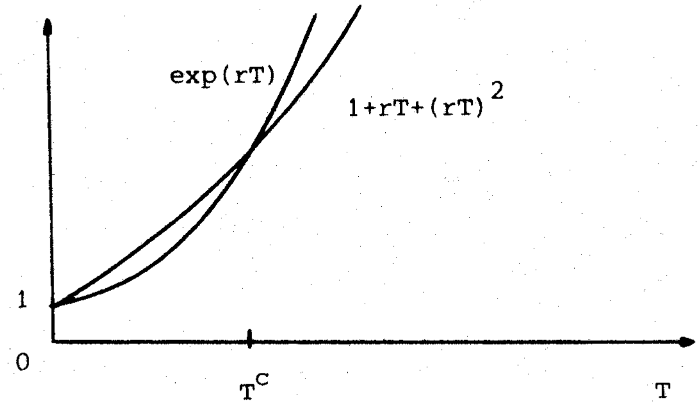


FIGURE 5

Now recall (Figure 4) that T^* is everywhere increasing in S_0 for given r and q, and everywhere increasing in q for given r and S_0. Further, by (22), it is always possible to find some small (large) enough S_0 such that $T^* < (>) T^c$, with equality occuring at only one value of S_0, given the values of r and q. Similarly (by (23) in the case of q.

It remains to establish the results about the pace of depletion. First, choose S_0 and q such that $T^* < T^c$, and thus $\partial K/\partial r > 0$, and consider a small increase in the rate of interest from r_1 to r_2. Figure 6.1 illustrates the effect of this on the price path. Over the initial capacity constrained phase (which may be longer or shorter than before), price is now lower. But price grows faster (at r_2 versus r_1 per cent) over the later phase. It is clear that, up to the date T' at which the profiles intersect, cumulative depletion is greater (and so the remaining stock smaller) on profile 2 than on profile 1. Moreover, for t>T' $D(p^2(t)) < D(p^1(t))$; therefore

$$NS^2(t) = \int_t^\infty D(p^2(\tau))d\tau < \int_t^\infty D(p^1(\tau))d\tau = NS^1(t).$$

That is, a small increase in the interest rate makes the depletion programme strongly less conservationist, as asserted.

Finally, choose S_0 and q such that $T > T^c$, or $\partial K/\partial r < 0$. Consider again a small increase in the interest rate from r_1 to r_2. Figure 6.2 shows the effect on the price path. Clearly profile 2 must intersect profile 1 at least once (at date T''). Otherwise total sales over time would be less along profile 2. Since p^2 eventually rises at a higher percentage rate than p^1 (r_2

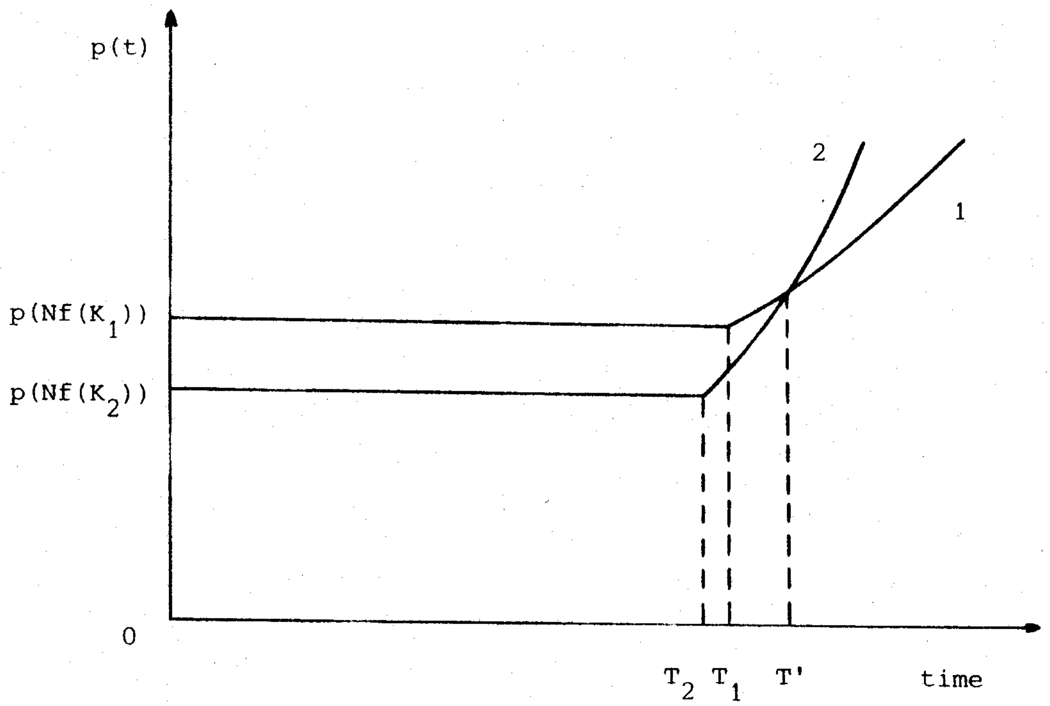


FIGURE 6.1

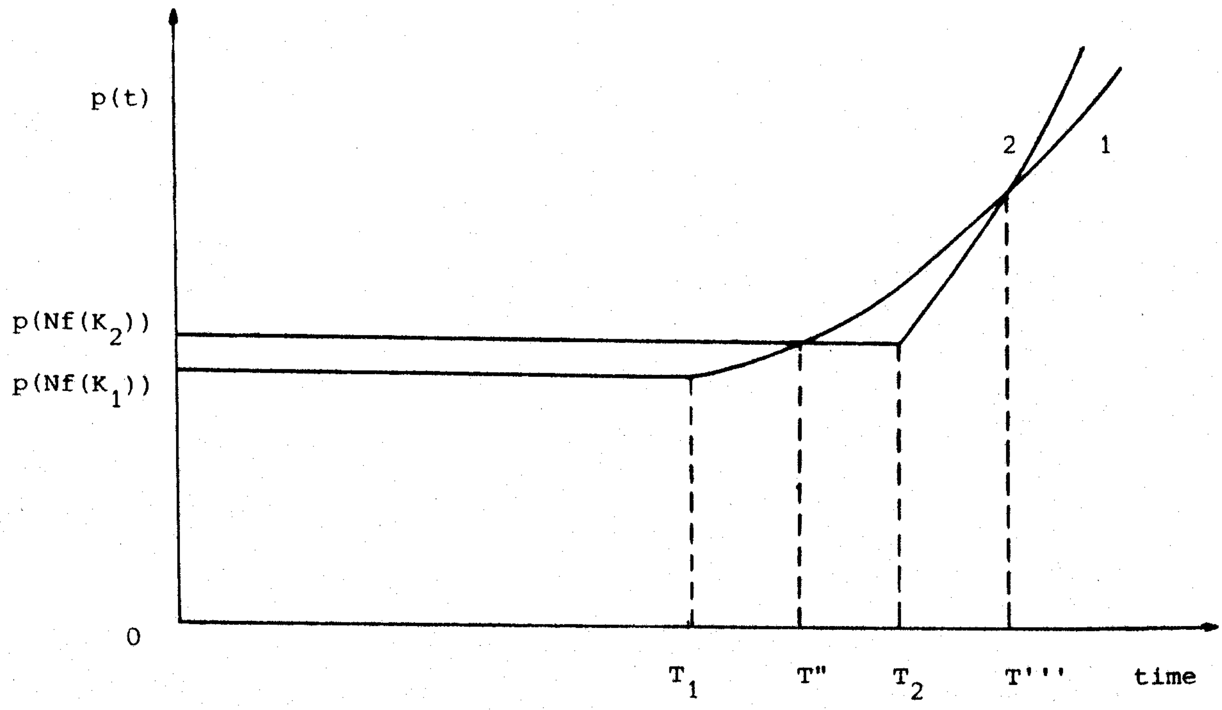


FIGURE 6.2

versus r_1) the paths must cross again at some date T'''. For $t \leq T''$

$$\int_0^t D(p^2(\tau))d\tau < \int_0^t D(p^1(\tau))d\tau,$$

so that profile 2 has larger remaining reserves than profile 1. On the other hand, for $t > T'''$,

$$NS^2(t) = \int_t^\infty D(p^2(\tau))d\tau < \int_t^\infty D(p^1(\tau))d\tau = NS^1(t),$$

so it is profile 1 that has the larger remaining reserves. Thus, in this case, a small increase in the interest rate implies greater conservation, but only in the weak sense. □

Roughly speaking, the result in propositions 1 and 2 can be rationalized as follows: A higher interest rate means, cet. par., that the net benefit stream should be tilted towards earlier dates. Where extraction is costless - or does not require capital - this simply means that more of the resource should be extracted early in the programme. To do this here requires that additional capacity be built up, which is costly. The gain from increasing resource use in earlier periods must therefore be balanced against the loss associated with the capacity increase that this requires.

If the resource is sufficiently scarce relative to capital (where the scarcity of the resource is captured by the size of S_0 and capital by the magnitude of q), and therefore comparatively highly valued, the gain outweighs the loss. A larger capacity is chosen, and more of the resource is extracted initially. On the other hand, if the resource is very abundant relative to capital, the loss exceeds the gain and it pays on the contrary to pick a smaller level of capacity, so less of the resource is used up initially.

3. EQUILIBRIUM UNDER COMPETITIVE EXPLOITATION

This section verifies that the optimal allocation and capacity buildup under social management can be reproduced - under suitable assumptions - if the resource deposits are competitively exploited. Though perhaps obvious, this result is worth demonstrating; it emphasizes that the installation costs, although they are borne at the outset, are not fixed costs in the sense that they give rise to non-convexities. By contrast, Hartwick et al. (1986) show that, if a fixed setup cost has to be incurred before a given deposit can be exploited (but there is no upper bound on the extraction rate thereafter), a competitive equilibrium fails, in general, to sustain the optimal extraction programme.

Assume then that each of the N deposits is unitized[10] and controlled by a price-taking operator. Assume also that the market rate of interest correctly reflects the social rate of discount. The following assumption is also used:

(A.6) There is a complete set of forward markets in which all future transactions in the resource are concluded at the initial date.[11]

Under these circumstances, given the equilibrium resource price trajectory $\{p(t)\}_{t=0}^{\infty}$, the operator with the rights to deposit i will choose an investment and extraction policy to maximize

$$(24) \quad \int_0^{\infty} e^{-rt}\{p(t)R_i(t) - qI_i(t)\}\, dt$$

subject to constraints (2)-(5). It is easily checked that a solution that maximizes (24) must satisfy (6)-(8) (with μ_i, the multiplier function corresponding to the resource stock constraint (2), constant over time) as well as (2)-(5) and the boundary condition on λ_i. For each individual firm, the resource price is parametric, but of course in the aggregate $p(t) = p(\Sigma_i R_i(t))$ at date t.

The set of conditions that characterize a competitive equilibrium is therefore the same as the one under social management. An optimal programme can thus in principle be decentralized.[12]

In the case of identical deposits (assumption (A.5)), all are exploited at the (same) maximum rate on an initial interval of time (0,T). On (T,∞), industry output declines so that the resource price increases at the rate of r per cent. The firm exploiting deposit i therefore chooses K_i^* as the solution to

$$(25) \quad \max_{K_i} \left\{ \left(\frac{1-e^{-rT}}{r}\right) \bar{p}\, f(K_i) + e^{-rT} \bar{p}\, S_i(T) - qK_i \right\},$$

where

$$(26) \quad S_i(T) = S_0 - Tf(K_i),$$

treating $\bar{p}$ and T as parametric. Using (26) in (25), the solution $K_i^* = K^*$ is seen to be independent of i. Thus $\bar{p} = p(Nf(K^*))$ and the equation defining K^* is precisely (14'):

$$(14') \quad \frac{(1 - (1+rT)e^{-rT})}{r} p(Nf(K))f'(K) = q.$$

This is referred to frequently in the next section as the benchmark condition for capacity choice, relative to which distortions are assessed.

Finally, every deposit is exploited at maximum capacity until at least date T. After T, operators are indifferent about how the extraction of reserves that remain in their deposits should be spread out over time, and the allocation of total production across fields is not determinate.[13]

4. DISTORTIONS FROM THE OPTIMAL PROGRAMME

This section deals with sources of bias from the social optimum (or perfectly competitive equilibrium). The first subsection examines the distortions attributable to commonly used tax instruments. The second and third subsections deal with the implications of removing the assumption of competitive extraction. To fix ideas, both assumption (A.5) (identical deposits) and assumption (A.6) (forward transactions) are retained throughout this section.

A. Tax Instruments and their Effects

Much of the literature that deals with the effects of taxes on the pace of depletion uses the assumption that the (variable) unit cost of extraction is constant (Dasgupta and Heal, 1979, Ch. 12; Dasgupta, Heal and Stiglitz, 1980; Conrad and Hool, 1981). Other work has incorporated more general cost conditions (Heaps, 1985; Gaudet and Lasserre, 1983), and arrives at broadly similar results about the bias introduced by various taxes. A number of these results can also be confirmed in the present context of irreversible capacity buildup.

For convenience, the usual technique of dealing piecemeal with the effects of different fiscal instruments is adopted here. The method will be to gauge the effect of each on a competitive operator's capacity choice.[14]

A tax on operating profits. It is well known that, where no fixed factors are used in the extraction process and the tax rate is time-invariant, this tax is equivalent to a rent tax and is therefore neutral. That is, it preserves the tax-free equilibrium (Dasgupta, Heal and Stiglitz, 1980). However, in the present case, it induces a distortion in the form of overconservation. To see this, consider the benchmark equation for capacity choice (14'). An operating profits tax at rate $0<\gamma<1$ changes the equilibrium condition to

$$(27) \quad (1-\gamma)(1-(1+rT)e^{-rT})p(Nf(K))f'(K) = q,$$

and it is clear that the solution of (27), say K', satisfies $K'<K^*$ (in view of (16), T is a decreasing function of K, and $K'\geq K^*$ with $T'<T^*$ would violate equation (27)). The tax therefore reduces total production of the resource on an initial interval. In fact, the taxed equilibrium is strongly more conservationist than the untaxed one.[15] The larger the tax rate, the greater is the magnitude of overconservation.

A tax writeoff provision for capacity installation costs. A profile on which operating profits are taxed can, however, be kept neutral. This requires a provision that permits operators to offset immediately <u>all</u> capital expenditures against tax liability, and is equivalent to subsidizing all such expenditures at the same rate as the per-unit tax on operating profit. To clarify the equivalence, note that capital costs incurred in the exploitation of deposit i at the initial date (qK_i) can also be expressed as a perpetual stream to the amount rqK_i at each date (interest payments or forgone interest). The writeoff provision makes the effective installation cost $(1-\gamma)qK_i$, and the equilibrium condition for capacity choice in deposit i coincides with the benchmark case.[16]

A revenue depletion allowance. A depletion allowance is a provision that allows the operator to deduct a certain amount from taxable income to compensate for the reduction in reserves that current production from the deposit entails. The allowance may be of the per-unit output type (see below), or it may permit the operator to deduct a given proportion ϕ $(0<\phi<1)$ of current revenue. Suppose the latter is combined with a time-invariant rent tax at rate γ. In this case, net returns to extraction from deposit i at any given date t are

$$(28)\quad \{p(t)R_i(t) - rqK_i) - \gamma(p(t)R_i(t) - rqK_i - \phi p(t)R_i(t)\}$$

$$= (1-\gamma)\{p(t)R_i(t)\left(1 + \frac{\gamma\phi}{1-\gamma}\right) - rqK_i\}.$$

The net present value of the deposit is then

$$(1-\gamma)\{\int_0^T e^{-rt}\left(\bar{p}f(K_i)\left(1 + \frac{\gamma\phi}{1-\gamma}\right) - rqK_i\right)dt$$

$$+ \int_T^\infty e^{-rt}\left(\bar{p}\, e^{r(t-T)}R_i(t)\left(1 + \frac{\gamma\phi}{1-\gamma}\right) - rqK_i\right)dt\}$$

$$= (1-\gamma)\{1-(1+rT)e^{-rT})\left(1 + \frac{\gamma\phi}{1-\gamma}\right)\bar{p}f(K_i) - qK_i + e^{-rT}\frac{\bar{p}(1-\gamma\phi)S_0}{1-\gamma}\},$$

where $\bar{p}=p(Nf(K))$ is the equilibrium price on $(0,T)$. Note that the r per cent rule is satisfied for $t>T$, because per-unit operating profit - the present value of which needs to be constant in equilibrium - is proportional to the resource price. Here the choice of K_i solves

$$(29)\ \ (1-(1+rT)e^{-rT})\ (1 + \frac{\gamma\phi}{1-\gamma})\ \bar{p}f'(K_i) = q.$$

Compare (29) with (27). It is clear that the depletion allowance is equivalent to a negative operating profits tax. It therefore induces overinvestment and (strong) underconservation. From (29), it is easy to verify that for the depletion allowance to be neutral, it must be applied to operating profit less "current" capital costs, rqK_i.

A severance tax. This is equivalent to a fixed fee per unit of the resource extracted. Denote this fee by γ. Then the operating profits of deposit i at date t are $\{p(t)-\gamma\}R_i(t)$.

Now recall that T has been defined such that on (T,∞) operators are indifferent about exactly when to extract. It then follows that the no-arbitrage condition

$$(30)\ \ e^{-rT}\ \{p(Nf(K_i)) - \gamma\} = e^{-rt}(p(t) - \gamma)$$

must hold for t>T. Using this fact, the present discounted value of deposit i is given by

$$(31)\ \ \left(\frac{1-e^{-rT}}{r}\right) \{\bar{p}(Nf(K_i)) - \gamma\}\ f(K_i)$$

$$+\ \ e^{-rT}\{\bar{p}(Nf(K_i)) - \gamma\}\ S_i(T) - qK_i .$$

Substituting for $S_i(T)$ from (26), and maximizing (31) with respect to K_i (but taking $\bar{p}(Nf(K_i))$ and T as given) yields that

$$(32)\ \ (1-(1+rT)e^{-rT})\{\bar{p}(Nf(K_i)) - \gamma\}\ f'(K_i) = q.$$

Now (32) must imply a smaller value of K_i, say K_i'', than K_i^*, the solution to (25) where there is no tax in place.

Suppose instead that $K_i'' \geq K_i^*$. Then we must have $T'' < T^*$; otherwise the resource price would be at least as low along the taxed equilibrium as along the no-tax equilibrium, and strictly lower beyond a certain date. This is so because, from (30), p" rises at less than r per cent after T". p*, on the other hand, rises at the rate of r per cent after T*. So if $K_i'' \geq K_i^*$, implying that $p'' \leq p^*$ at the initial date, and $T'' < T^*$, total extraction is greater on the taxed profile than on the no-tax profile. Since total reserves are the same in both cases, this is a contradiction.

Going back to (32), it can now be noted that if K_i^* and T^* solve (32) when $\gamma = 0$, then K_i'' and T'' with $K_i'' \geq K_i^*$ and $T'' < T^*$ cannot solve (32) when $\gamma > 0$. In short, $K_i'' < K_i^*$, and the presence of a severance tax at rate γ brings about overconservation in the strong sense.

An output depletion allowance. This is a provision that allows the operator to deduct a fixed amount (denoted by ϕ) per unit of the resource extracted from taxable income. Suppose then that this provision is combined with a time-invariant <u>rent</u> tax at rate γ. Rent captured by the operator from deposit i at date t is given by

$$(p(t)R_i(t) - rqK_i) - \gamma\{p(t)R_i(t) - rqK_i - \phi R_i(t)\}$$

$$= (1-\gamma)\{(p(t) + \frac{\gamma\phi}{1-\gamma})R_i(t) - rqK_i\}.$$

This indicates that the effect of the depletion allowance is equivalent to that of a negative severance tax. Accordingly, an argument exactly symmetric to the one used above can be adduced to show that the profile is strongly less conservationist than the benchmark profile. Price grows at a rate larger than r after at least one deposit ceases to operate at maximum capacity.[17]

To conclude this subsection, two points are in order. The first is that even if a tax does not affect the rate of price increase (r per cent) that prevails when the capacity constraint no longer bites, as in the case of the operating profit tax, it will still distort the extraction profile unless appropriate allowance is made for capital expenditure to be written off against tax. The second point is related: if an operating profits tax is imposed on operators <u>after</u> they have installed all the capital equipment (on the assumption that no such tax - or a lower rate - would obtain), then, unlike a severance tax, it would not distort the depletion profile. Of course, to the extent that deposits are located, developed and exploited in overlapping sequence rather than simultaneously, springing operating profit taxes on current fields inhibits the development of new ones if operators anticipate that this will recur in the future.[18]

B. Monopoly

This subsection considers the distortions inherent to monopoly. To focus on these alone, the assumption is retained that there is a complete set of forward markets at the initial date (assumption (A.6)). For the monopolist in control of N resource deposits who seeks to maximize the present value of profits, necessary and sufficient conditions for a maximum (analogously with conditions (2)-(8) and the boundary conditions on the λ_i's) are

(33) $e^{-rt}\{p'(R)\Sigma_i R_i + p(R)\} \geq \mu_i$, $R_i \leq f(K_i)$ (CS)

and $e^{-rt}\{p'(R)\Sigma_i R_i + p(R)\} < \mu_i$ implies $R_i=0$;

(34) $e^{-rt}q \geq \lambda_i$, $I_i \geq 0$ (CS); and

(35) $\dot{\lambda}_i = -\{e^{-rt}(p'(R)\Sigma_i R_i + p(R)) - \mu_i\}f'(K_i)$,

and (2)-(5), with μ_i constant for $t \geq 0$ and $\lim_{t \to T_i} \lambda_i(t)=0$, $i=1,..,N$.[19] (CS) denotes that the inequalities hold with complementary slackness. Recall also that $R = \Sigma_i R_i$. Arguments along the lines of those used in Appendix A can be used to show that, for each deposit, capacity installation is instantaneous at t=0 and remains unchanged thereafter. Moreover, under assumption (A.5), equilibrium capacity is precisely the same in every deposit.

The story is much the same as previously: for an initial period (0,T) all deposits are exploited at maximum capacity. On (T,∞) there is at least one deposit for which output is positive but below capacity at any given date. Aggregate extraction is adjusted so that marginal revenue to the monopolist appreciates at rate r (see (33) with $R_i < f(K_i)$). The resource price is continuous at all dates.

How does the monopolist's capacity choice differ from that of the competitive industry, and what is the effect on the pace of resource use? As might be expected, the answer depends, ceteris paribus, on the properties of the demand function. Specifically, it depends on the behaviour of the elasticity of demand, here denoted by $\eta(R)=p(R)/p'(R)R$. To ensure a positive level of output, it is necessary to assume that $\eta(R)<-1$.[20] It will prove useful to isolate three prototype cases for analysis: (i) $\eta'(R)=0$; (ii) $\eta'(R)<0$; and (iii) $\eta'(R)>0$, in each case over the entire equilibrium range of output. Each of the three cases is treated in turn.

Case (i): $\eta'(R)=0$. The isoelastic case is the simplest to analyze. It is also a benchmark case in that, where there are no variable (or fixed) costs of extraction, the monopoly equilibrium is distortion-free (Stiglitz, 1976, and Dasgupta and Heal, 1979, pp. 325-7). That the elasticity is constant implies that marginal revenue, $p(1+1/\eta)$, is proportional to the resource price. Since for $t \geq T$ marginal revenue grows at the percentage rate r, so does price. The discounted revenue stream accruing to the monopolist is then given by

$$(36)\quad V^M = \int_0^T e^{-rt} p(Nf(K))Nf(K)dt + e^{-rT} p(Nf(K)) \int_T^\infty D(p(Nf(K))e^{r(t-T)})dt$$

where D(.) is the inverse function of p(.). (36) is subject to the reserves constraint

$$(37)\ NS_0 = NTf(K) + \int_T^{\infty} D(p(Nf(K))e^{r(t-T)}dt\ .$$

Using (37) in (36), the latter becomes

$$(36')\ V^M = N\left[\frac{p(Nf(K))f(K)}{r}(1-(1+rT)e^{-rT}) + e^{-rT}p(Nf(K))S_0\right]$$

It is now straightforward, albeit somewhat laborious (see Appendix B, part (i)) to establish that the derivative of (36') with respect to K reads simply

$$(38)\ V^M_K = N\left[(1-(1+rT)e^{-rT})\frac{p(Nf(K))f'(K)}{r}(1+\frac{1}{\eta})\right]$$

Thus, for given K, $V^M_K < V^*_K$ (see equation (14')), where the superscripts refer to the outcomes under monopoly (or cartel) management and social management (or competitive exploitation) respectively. Since both V^M_K and V^*_K are equated with Nq and moreover $V^M_{KK}, V^*_{KK} < 0$, it follows that $K^M < K^*$.

Figure 7 compares the time-path of prices sustained by the monopolist with the competitive path. It is immediate that $p^M(T^M) < p^*(T^M)$, where T^M is the date after which price begins to grow at the rate of r per cent under monopoly. Failing this, at least one of the profiles would violate the stock exhaustion condition (37) (note that (37) is identical to (13)). Strong overconservation under monopoly can be shown using an argument symmetric to the one for propositions 1 and 2, section 2C.

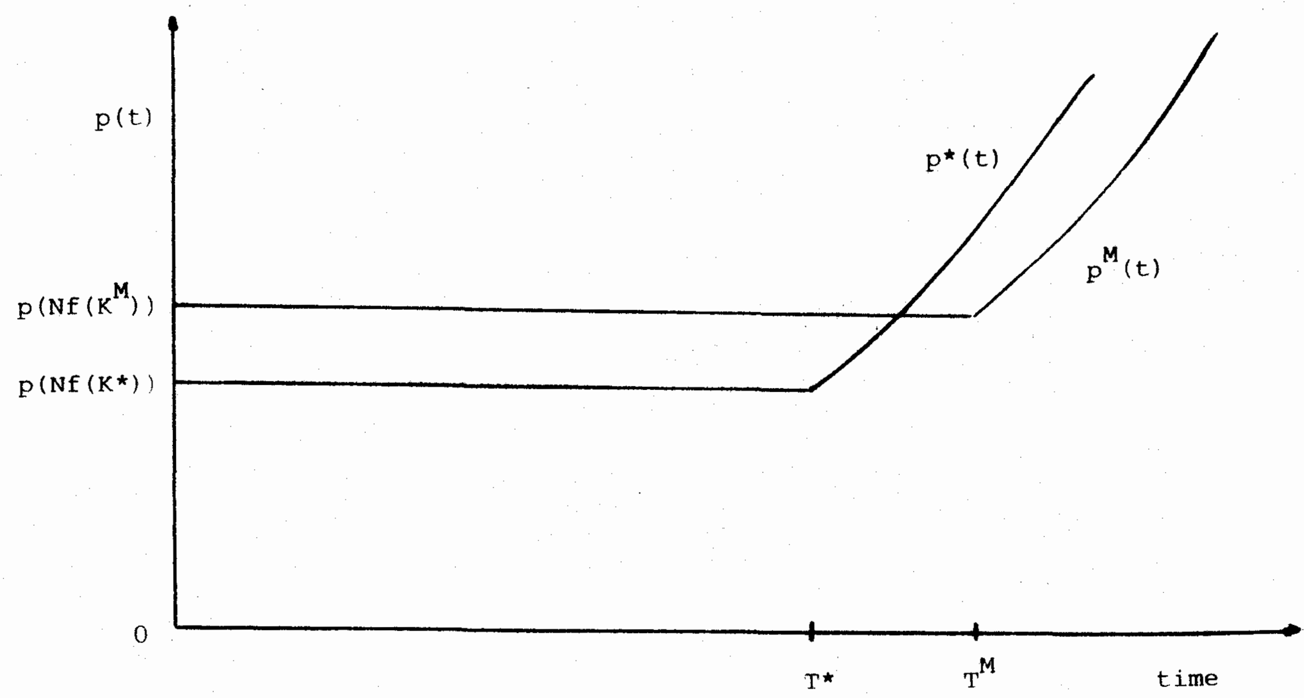


FIGURE 7

The result is summarized in proposition 3:

Proposition 3 Under isoelastic demand, a monopolist exploits its resource deposits at suboptimal scale (from a social viewpoint). Over an initial interval of time the resource is overpriced, and the resource stock is strongly overconserved.

The result is not, on reflection, surprising. It is true that all costs are fixed costs ("up front" irrecoverable costs as opposed to "quasi-fixed" costs that are incurred per period only so long as output is positive). Under isoelastic demand, these would not in themselves be expected to introduce a wedge between the monopoly and competitive extraction paths.[21] But here an increment in fixed costs is the cost of purchasing a unit of capital at the outset. In equilibrium, this is equated with the return to an increment in scale. The latter is, loosely speaking, proportional to the return from the sale of a small unit of output at the initial date, when output from each deposit is capacity constrained. For a given scale of operation (hence a given resource price) marginal revenue lies below price, so the return to the sale of this incremental unit is less for the monopolist than for the competitive supplier. The monopolist therefore stops short of the competitive scale of operation.

Case (ii): $\eta'(R)<0$. This is the case where the elasticity of demand increases in absolute value as output of the resource increases. Dasgupta and Heal (1979, pp. 327-8) and Lewis et al. (1979) argue that this case is plausible and may arise if a larger equilibrium output (and lower price) means that the resource comes to be used in industries that can easily switch back to substitute inputs. "Marginal" demand thus dissipates very quickly with even a small increase in price.

From the optimality condition (33), if there is at least one deposit for which capacity is not fully employed, $e^{-rt}p(R)(1+1/\eta(R))$ must be constant over time (recall that $R>0$ always because there is no chokeoff price). Time-differentiating and rearranging (33) then gives

$$\frac{\dot{p}}{p} - \frac{\eta'(R)\dot{R}}{\eta(R)(1+\eta(R))} = r,$$

implying (since $\dot{R} < 0$) that the monopolist wishes to sustain $\dot{p}>rp$ for $t>T$ when the capacity constraint no longer bites. But unless the resource is instantly perishable, or its storage costs - except in the ground - prohibitive, such a price trajectory cannot support asset market equilibrium. Everybody would attempt to buy up stocks of the resource, because the resource now yields a higher return than the numeraire asset does. In short, the monopolist faces the constraint that $\dot{p}/p \leq r$ at all dates.

Thus for t>T the constraint bites and price grows at the rate of r per cent. So (36') measures the present value of revenue accruing to the monopolist. As shown in Appendix B, part (ii), the derivative of (36') with respect to K in the case where $\eta'(R)<0$ is given by

$$(39)\quad V^M_K = N\left[(1-(1+rT)e^{-rT})\frac{p(Nf(K))f'(K)}{r} + \left\{1-e^{-rT}\left(1+\frac{rT}{G(T)}\right)\right\}\frac{p(Nf(K))f'(K)}{r\,\eta(Nf(K))}\right],$$

$$\text{where } 0 < G(T) = \frac{\int_T^{\infty}\eta(D(p(Nf(K))e^{r(t-T)}))D(p(Nf(K))e^{r(t-T)})dt}{\eta(Nf(K))\int_T^{\infty}D(p(Nf(K))e^{r(t-T)})dt} < 1$$

and, in equilibrium, K^M solves $V^M_K = Nq$. Again using equation (14'), it transpires that for given K, $V^M_K \geq V^*_K$ if and only if $e^{rT} \leq 1+rT/G(T)$.

In sum, the direction of bias for the case $\eta'(R)<0$ depends on the magnitude of G(T). Suppose for example that G(T) is sufficiently below unity (that is, $\eta(R)$ rises - or the elasticity falls in absolute value - at a sufficiently fast rate as output declines during the final interval (T,∞)). Then it is possible that, for given K, $V^M_K>V^*_K$. Since both are equated with Nq in equilibrium, and both are decreasing functions of K, $K^M > K^*$. Per-field capacity choice is greater under monopoly than under competition. It follows that monopoly extraction is overly profligate in the strong sense.

Conversely, if G(T) is close to unity, $V^M_K < V^*_K$ and the reverse applies. What is interesting, though, is that in the present context monopoly can in principle be associated with excessive (strong) profligacy, even if speculative purchases and storage of the resource are feasible. Speculation prevents excessively rapid depletion in the costless extraction model, because it thwarts the more rapid price growth that is required to tilt the production schedule in favour of earlier periods (see Dasgupta and Heal, 1979, pp. 329-31). In the present case, of course, extraction is simply tilted in favour of the present through a slightly larger initial capacity buildup.

Case (iii): $\eta'(R)>0$. Here demand is assumed to become more elastic (that is, η declines) as the size of the market contracts. Dasgupta and Heal (1979, p.328) state the case for this:

> ..... as the price is raised, this increases the incentive to invent substitutes that did not previously exist, or to proceed with development work on potential substitutes whose development has been held in abeyance while the resource price was low.

Where extraction is costless, the case $\eta'(R)>0$ is associated with (strong) overconservation. It is straightforward to establish that this is so in this context too. Denote marginal revenue, evaluated at R>0, by $m(R)=p(R)(1+1/\eta(R))$. Differentiating this gives

$$m'(R) = p'(R)(1+1/\eta(R)) - p(R)\eta'(R)/\eta(R)^2.$$

Since $\eta'(R)>0$, $m'(R)<0$ over the output range under consideration, and m(.) may therefore be inverted.[22] Let n(.) be its inverse. Now for $t \geq T$,[23]

$$(40)\quad m(R(t)) = e^{r(t-T)}\, m(Nf(K))$$

that is, the percentage growth rate of marginal revenue is r. This follows from (33) and the fact that, by definition, at least one deposit is no longer exploited at maximum capacity after T. (40) then implies that, for $t \geq T$, $R(t) = n(m(Nf(K))e^{r(t-T)})$, so that $p(R(t)) = p(n(m(Nf(K))e^{r(t-T)}))$. Thus the present value of revenue received by the monopolist can be written

$$(41)\quad V^M = (1-e^{-rT})\,\frac{p(Nf(K))Nf(K)}{r}$$

$$+ \int_T^\infty e^{-rt}p(n(m(Nf(K))e^{r(t-T)}))n(m(Nf(K))e^{r(t-T)})dt.$$

(41) is of course defined subject to the resource stock constraint

$$(42)\quad NS_0 = NTf(K) + \int_T^\infty n(m(Nf(K))e^{r(t-T)})dt.$$

The derivative of (41) with respect to K - treating T as an implicit function of K given by (42) - is somewhat lengthy to compute. The steps are given in Appendix B, part (iii) and the derivative is

$$(43)\quad V^M_K = N\,\{(1-(1+rT)e^{-rT})\frac{f'(K)m(Nf(K))}{r} + e^{-rT}p(Nf(K))f(K)\frac{dT}{dK}\}.$$

The solution (K^M,T^M) here solves $V^M_K=Nq$ and (42). To compare K^M

and K^*, recall that (K^*,T^*) is the solution to equations (14') and (13).

Suppose first that $K^M=K^*$. Then from the resource stock constraints (42) and (13), it is clear that $T^M<T^*$. That is, the resource price must begin to rise earlier under the present case than under competition. This is because price here grows at less than r per cent, so if it begins its growth no earlier than in the competitive case, it remains under the competitive price in perpetuity after T^M. Cumulative sales are consequently larger than under competitive exploitation. Since total reserves are the same in each case, either the competitive extraction profile violates (13) or the monopoly profile contradicts (42).

Now since $T^M<T^*$,

$$1-(1+rT^M)\exp(-rT^M) < 1-(1+rT^*)\exp(-rT^*),$$

and also $m(Nf(K))<p(Nf(K))$ and $dT/dK<0$. So for $K^*=K^M$ one infers from (43) and (14') that $V^M_K<V^*_K$. Since each of these must be equated with Nq, K^M must be lowered below K^* a little to raise V^M_K and equate it with V^*_K. Thus $K^M<K^*$.

In the case $\hbar'(R)>0$, then, the resource is unambiguously overpriced for an initial period of time, but underpriced later in the programme. It is easy to show that this implies a more conservationist depletion programme (in the strong sense) than under social management.

The results for cases (ii) and (iii) are summarized in proposition 4:

Proposition 4 Where $\eta'(R)>0$ over the relevant range of the demand curve, a monopolist builds up too little capacity, and the resource is overconserved in the strong sense. However, where $\eta'(R)<0$ over the relevant range, the direction of bias is in general indeterminate, but the possibility does arise that the monopolist will overinvest in capacity and underconserve in the strong sense.

Finally - as is more likely to be the case - where the behaviour of the elasticity of demand is a hybrid of the three prototype cases considered above, little can be said in general about the direction of bias. The preceding analysis does, however, give an indication of the various forces that operate. The direction of bias on the whole inclines towards excessive conservation.

C. Symmetrically Placed Oligopolists: the Cournot-Nash Solution

Pure monopoly and perfect competition are polar cases that seldom find application in practice. It is useful, therefore, to have some idea of the properties of equilibrium in "intermediate" market structures. A number of authors have tackled this problem in the context of exhaustible resource extraction. Their work tends to concentrate on two types of conjectural structure: Nash-Cournot quantity setting and Stackelberg leader-follower structures.[24]

In the latter case, a dominant supplier of the resource coexists with a fringe group that also supplies the resource or a conventionally produced substitute. The dominant supplier optimizes by dictating a price trajectory that already incorporates the (passive) fringe's known supply response. This class of problems has been treated by, among others, Gilbert (1978), Ulph and Folie (1981), Newbery (1981) and Stiglitz and Dasgupta (1982, section 6). Among the interesting results to emerge from this work, two in particular deserve mention. One is that the "limited competition" in this framework does not necessarily imply an outcome that lies "between" the outcomes under monopoly and competition. For example, the resource may be overpriced initially relative even to the pure monopoly equilibrium, which already overprices the resource vis-a-vis the competitive case.

A second result is that in general it pays the leader to announce an initial strategy which there is an incentive later to deviate from. This arises essentially because the process of extraction constitutes an <u>irreversible</u> diminution of stock. Once the fringe has been coaxed into extracting its entire stock, any source of competition disappears; so there is an incentive for the dominant supplier to engineer this through a fake announcement. The story is then carried a step further: the fringe recognizes the dominant supplier's incentive to fool it, and - in the absence of a commitment vehicle - assumes that it will do so, and responds accordingly. The appropriate equilibrium concept thus becomes one in which the equilibrium is computed recursively - a "subgame perfect" equilibrium - and differs substantially from a precommitment equilibrium.[25]

Because in the present context it is not only extraction but also the decision about capacity buildup that is irreversible, the problem of dynamic inconsistency would undoubtedly surface in a leader-follower problem. However, the lack of symmetry in the equilibrium would also appear to make its characterization difficult in the present context. For this reason the symmetric Cournot-Nash equilibrium is characterized instead.

Among the analyses of the precommitment (or open-loop) version of the Cournot-Nash equlibrium in the resource extraction context is that of Lewis and Schmalensee (1980). They show in particular that, in the case of identical (and constant) unit cost and reserve conditions, the larger the number of firms, the quicker the resource is used up (see also Dasgupta and Heal, 1979, pp. 336-40). Loury (1986) considers the case of identical constant unit costs but different reserves across firms, and demonstrates that if reserve differences are not too large (so that all firms exhaust together), the resulting extraction path maximizes a weighted average of profits and social surplus. From this he is able to show that the open-loop Nash-Cournot equilibrium is strongly overconservationist, with the extent of overconservation non-decreasing in the number of operators.

In a paper by Ulph and Folie (1980), a cartelized group of firms who behave as a Nash-Cournot player share the market with a price-taking fringe group. For the linear demand and extraction cost case, Ulph and Folie assume that the cartel has a significantly lower unit cost than the fringe. They prove that in this case the discounted profits of the fringe are lower than they would be under ubiquitous competition, and argue that their results extend to the convex costs case. This shows that Salant's (1976) result that a fringe group is better off under cartelization if cost and reserves conditions are uniform is not generally valid. In a slightly different context, Stiglitz and Dasgupta (1982, section 7) consider a duopoly equilibrium where one firm supplies the resource and the other (potentially) a conventionally produced substitute. Interestingly, they derive the novel proposition that in general an interval of simultaneous supply obtains. Usually the backstop is held in abeyance until the resource is depleted.

The question, however, is whether in the Cournot-Nash case the open-loop solution technique is seriously misleading in the absence of a vehicle - binding forward contracts, for example - whereby firms can commit themselves to an announced extraction policy for some period of time (the commitment period). For a common property resource, Reinganum and Stokey (1985) demonstrate that the answer is a definite "yes". Using a costless extraction and isoelastic demand framework, they parametrize the length of the commitment period and show that the speed of extraction is a declining function of it. Moreover the resource stock is used up arbitrarily fast as the length of the commitment period tends to zero. This is an intuitively appealing result, but it appears to be a direct result of the common property assumption. Eswaran and Lewis (1985) find that under private ownership, open-loop and recursive solutions coincide in simple cases (such as isoelastic demand and costless extraction for all firms), and appear to be remarkably close in other cases. Though by no means conclusive, their results suggest that the problem of dynamic inconsistency may not be a serious one in the private ownership Cournot-Nash case, and provide some justification for restricting attention here to the open-loop solution concept.

To proceed: each firm is assumed for convenience to have the exclusive right to one deposit. N therefore denotes the number of operators as well as the number of deposits. Deposits are assumed identical (assumption (A.5)). Each operator is a PV profit maximizer and plans its entire time-profile of extraction at the initial date, taking the extraction policies of other firms as given. Firm i's choice $\{R_i(t), I_i(t)\}_{t=0}^{\infty}$ must therefore satisfy the following conditions:

$$(44)\quad e^{-rt}\Big[p'\big(\sum_{j\neq i}\bar{R}_j(t)+R_i(t)\big)R_i(t) + p\big(\sum_{j\neq i}\bar{R}_j(t)+R_i(t)\big)\Big] \geq \mu_i, \qquad 0<R_i(t)\leq f(K_i(t)) \quad \text{(CS)}$$

$$\text{and}\quad e^{-rt}\Big[p'\big(\sum_{j\neq i}\bar{R}_j(t)+R_i(t)\big)R_i(t) + p\big(\sum_{j\neq i}\bar{R}_j(t)+R_i(t)\big)\Big] < \mu_i \quad \text{implies } R_i(t)=0;$$

$$(45)\quad e^{-rt}q \geq \lambda_i(t), \qquad I_i(t) \geq 0 \ (CS);$$

$$(46)\quad \dot{\lambda}_i(t) = -\{e^{-rt}[p'(\sum_{j\neq i} \bar{R}_j(t)+R_i(t))R_i(t) + p(\sum_{j\neq i} \bar{R}_j(t)+R_i(t))] - \mu_i\} f'(K_i(t));$$

$$(47)\quad \int_0^{\infty} R_i(t)dt = S_0$$

$$(48)\quad \dot{K}_i(t) = I_i(t);\ K_i(0)=0$$

where μ_i is constant, i=1,..,N. A bar above a variable indicates its assumed constancy to firm i under its conjectures. (44) states that the operator should produce at maximum capacity if the revenue from the sale of an incremental unit exceeds the user cost of the resource, and should produce nothing if it is less. If the two are equal, the operator is indifferent about its output rate. Similarly, (45) states that it is worth adding to the capital stock if the stream of discounted returns to doing so exceeds the purchase cost, and (46) states that each component of the stream is the net marginal revenue product of capital. Finally, (47) just says that firm i's reserves will be used up in the long run.

An intertemporal equilibrium $\{R_i(t), I_i(t)\}^{\infty}_{t=0}$, i=1,..,N, is defined such that the strategies of all N operators are mutually compatible. The solution in this instance is simple. All firms adjust to desired capacity at the initial date and maintain that level thereafter. For an initial period (0,T), capacity is fully utilized by every operator.

Suppose now that demand is isoelastic. Then on the subsequent interval (T,∞) total output declines smoothly so that the resource price grows at the rate of r per cent. In fact on this interval of time $R_i(t)=(1/N)R(t)$, $i=1,..,N$. That is, total output is symmetrically distributed across firms.[26] The PV of operator i's revenue stream is then

$$(49)\quad V_i = \left(\frac{1-e^{-rT}}{r}\right) p((N-1)f(\bar{K})+f(K_i))f(K_i)$$
$$+ e^{-rT}\, p((N-1)f(\bar{K})+f(K_i))\{S_0-Tf(K_i)\}$$

Note that although $K_i=\bar{K}$ in equilibrium, the two are kept conceptually separate, since firm i takes others' scale of operation to be independent of its own decisions. $\underset{K_i}{\mathrm{Max}}\{V_i-qK_i\}$ now provides the rule for choosing K_i. Using (49), firm i therefore chooses K_i to satisfy equation (50):

$$(50)\quad \left(\frac{1-(1+rT)e^{-rT}}{r}\right)f'(K_i)\{p(.) + p'(.)f(K_i)\}$$
$$+ rTe^{-rT}p(.)f(K_i)\frac{dT}{dK_i} + e^{-rT}\, S_0\{p'(.)f'(K_i) - rp(.)\frac{dT}{dK_i}\} = q,$$

where the argument of p(.) and p'(.) is understood to be $(N-1)f(\bar{K})+f(K_i)$. The derivative dT/dK_i is found by implicit differentiation from the firm's reserves constraint (51):

$$(51)\quad S_0 = Tf(K_i) + \frac{1}{N}\int_T^{\infty} D(p((N-1)f(\bar{K})+f(K_i))e^{r(t-T)})dt.$$

Substituting the derivative back into (50), a little manipulation (following the steps in Appendix B, part (i)) shows that the equilibrium condition can be written as

$$(50')\quad \left(\frac{1-(1+rT)e^{-rT}}{r}\right) p((N-1)f(\bar{K})+f(K_i))\; f'(K_i)\; (1 + \frac{1}{N\eta}) = q.$$

Recall now that the conditions for the choice of K_i in the monopoly and competitive cases are given respectively by

$$(38') \quad N\,(1 - (1+rT)e^{-rT})\,\frac{p(Nf(K))f'(K)}{r}\,(1+\frac{1}{\eta}) = Nq \quad \text{and}$$

$$(14') \quad N\,(1 - (1+rT)e^{-rT})\,\frac{p(Nf(K))f'(K)}{r} = Nq \quad .$$

It is straightforward to confirm that, for given N, the solution here lies between the competitive and monopoly solutions. Per-field capacity choice is larger than under monopoly but smaller than under competitive exploitation. However, in comparing these cases it is important to bear in mind that in the present construct the number of operators cannot be changed without the number of deposits being changed at the same time. Recall also that each deposit contains a fixed amount of the resource, S_0.

If N is close (equal) to one, the term $(1+1/N\eta)$ is close (equal) to $(1+1/\eta)$, which arises in the monopoly case (see equation (38') with N close (equal) to one). Setting the number of firms equal to one thus retrieves the monopoly solution with one deposit. Conversely, suppose N is a large number. Then $(1+1/N\eta)$ is close to unity. The solution in this case coincides approximately with the competitive solution (see equation (14')) with a large number of deposits.

The result is summarized in proposition 5:

Proposition 5 Under isoelastic demand, a Nash-Cournot allocation overconserves the resource (in the strong sense) relative to the competitive outcome, but the magnitude of the distortion is smaller than under monopoly. If the number of operators is sufficiently large, the bias is negligible.

5. VARIANTS OF THE BASIC MODEL

How robust are the results derived in the basic model of section 2? Among the assumptions employed there are (a) the assumption that capital equipment can be purchased and installed at constant unit cost when so desired; (b) the assumption that no variable inputs are used in the extraction process; and (c) the assumption that the capital equipment is not subject to any physical depreciation whatsoever. The subsections that follow relax each of these in turn, and give an indication of how this modifies the basic properties of the solution characterized in section 2A.

A. Adjustment Costs in Capacity Installation

Suppose that the adjustment cost function for capacity installation in deposit i is given by $C_i(I_i)$. $C_i(.)$ is assumed to display the following properties:

(A.7) $C_i(.)$ is twice-continuously differentiable with $C_i(I_i)>0$, $C_i'(I_i)>0$ for $I_i>0$, $C_i(0)=C_i'(0)=0$, and $C_i''(I_i)>0$ for $I_i \geq 0$, $i=1,..,N$.

In words, capacity expansion entails increasing marginal adjustment costs. The adjustment cost function has been defined only for $I_i \geq 0$ because the assumption that the equipment has no resale value is retained. Assumption (A.7) now supplements assumption (A.4) in section 2.

Under the present construct, decisions about capacity buildup and extraction cannot be separated, even in the special case of identical deposits. Formally, the planner chooses time profiles $\{R_i(t), I_i(t)\}^{\infty}_{t=0}$, $i=1,..,N$ to maximize

$$(52) \quad \int_0^{\infty} e^{-rt}\{u(\Sigma_i R_i(t)) - \Sigma_i[qI_i(t)+C_i(I_i(t))]\}dt$$

subject to constraints (2) - (5), where $K_i(0)$ is assumed to be positive but "sufficiently small", $i=1,..,N$. The necessary and sufficient conditions for a maximizing solution are the same as those in section 2A (that is, (2)-(6) and (8) with μ_i constant, $i=1,..,N$). The only difference is that the optimality condition (7) now becomes

$$(53) \quad e^{-rt}\{q + C'(I_i(t))\} \geqq \lambda_i(t), \; I_i(t) \geqq 0 \quad (CS)$$

where $\lim_{t\to\tau_i} \lambda_i(t)=0$, $i=1,..,N$. Recall that τ_i is the date at which deposit i definitively ceases production, $0<\tau_i\leqq\infty$.

What are the general properties of the solution in this case? To begin with, it is clear that "bang-bang" adjustment is infinitely costly, so any capacity expansion must be "staggered" over time. Next, it is useful to note that, at any given date, one of the following regimes must be the applicable one for deposit i:

Type 1 phase : $I_i>0$, $R_i=f_i(K_i)$;

Type 2 phase : $I_i=0$, $R_i=f_i(K_i)$; or

Type 3 phase : $I_i=0$, $R_i<f_i(K_i)$.[27]

For N=1, or where deposits are identical in every respect, it is straightforward to show that a type 1 phase (which exists provided initial stocks of capital equipment are "low") must be the first to occur, and cannot recur once it ends. This is because an interval on which $I_i>0$ which begins at some date $t_1>0$ can be shown to imply a time-increasing resource price, which raises a contradiction in the single or identical deposit case (Appendix C demonstrates that under identical deposits optimality dictates identical investment rates). Under these circumstances, there is an initial interval of time on which the resource price falls, followed by a period on which price is constant, followed in turn by a (final) period on which the Hotelling rule is satisfied.

In the case of heterogeneous deposits, it does not appear possible in general to establish that for any given deposit j, a type 1 phase must be the first to occur, and does not recur. However, if a type 1 phase is observed for deposit j (this will be the case if $K_j(0)$ is sufficiently small), it is clear firstly that investment must fall smoothly to zero towards the end of this phase. This follows from (53) and assumption (A.7). Secondly, this phase must be followed by a type 2 phase; that is, a transition directly from a type 1 phase to a type 3 phase can be ruled out. Suppose the contrary, and let T_j denote the date at which the type 1 phase ceases. Then by the continuity of $\lambda_j(t)$ we must have $e^{-rT_j}q=\lambda_j(T_j)$; or, integrating (8),

$$(54)\quad q = \int_{T_j}^{\mathcal{T}_j} e^{-r(t-T_j)}\{p(R(t)) - \mu_j e^{rt}\}f'(K_j(t))\,dt,$$

where again $\mathcal{T}_j$ is the date at which the exploitation of deposit j stops definitively. Thus $p(R(t))>\mu_j e^{rt}$ (which implies that deposit j is exploited at maximum capacity) on at least one interval of time after T_j. Let (t_0, t_1), $t_0 \geqq T_j$, $t_1 < T_j$, denote the earliest such interval of time. It remains now to establish that $t_0 = T_j$; that is, capacity continues to be fully employed for some time after T_j. Suppose instead that $t_0 > T_j$; that is

$$p(R(t)) - \mu_j e^{rt} = 0, \quad t \in (T_j, t_0).$$

But then, since

$$p(R(t)) - \mu_j e^{rt} > 0, \; t \in (t_0, t_1),$$

p must (using the continuity of p and μ_j) grow at a larger rate than r per cent on a right-hand neighborhood of t_0. However, the fact that p is growing implies that there is at least one deposit - h, say - for which output is positive but declining over time. The optimality condition in turn implies that $p(R)-\mu_h e^{rt}=0$ on that neighborhood, and therefore contradicts the requirement that p should grow faster than r per cent. Thus $t_0 = T_j$; that is, for an arbitrary deposit j a type 1 phase must be followed by a type 2 phase, as asserted. If a type 3 phase arises for deposit j, a type 2 phase precedes it.[28]

In short, for the case where capacity expansion is subject to adjustment costs, it is difficult to say much in general about the time-distribution of investment profiles and output in individual deposits. If the initial stock of capital equipment in every deposit is sufficiently small, there is an initial interval of time on which the output rate in every deposit is non-decreasing and capacity is strictly increasing in at least one deposit.[29] In this case, one would observe a falling resource price for an initial period of time. Any period during which the resource price is rising, however, must display the Hotelling rule. The Hotelling rule must hold on a final interval of time, but a phase of r per cent price growth between periods of non-increasing prices cannot be ruled out. All that is required is that at least one deposit should exhibit a declining output rate during such a phase.[30]

B. A Variable Factor in the Extraction Process

The use in the extraction process of (at least) one variable input retrieves, under appropriate conditions, the result that the aggregate output rate is everywhere a declining function of time. To demonstrate this point, the construct is the same as in section 2A. Adjustment costs are once again ignored. The only difference is that constraint (4) here reads, for $t>0$,

$$(4') \quad R_i(t) \leq f^i(K_i(t), L_i(t))$$

(ignoring the non-negativity constraint on output). $f^i(.)$ is assumed to satisfy the following conditions

(A.8) $f^i(.)$ is twice-continuously differentiable and jointly (strictly) concave in both its arguments. Also $f^i(0,L)=f^i(K,0)=0$, $i=1,..,N$.

L is the variable factor, and has a flow price of w, for convenience assumed time-invariant. Note that (4') will always hold with equality; otherwise costs could be reduced by reducing L_i with no loss of output.

In order to maximize the Hamiltonian

$$H = e^{-rt}\{u(\Sigma_i f^i(K_i(t), L_i(t)) - q\Sigma_i I_i(t) - w\Sigma_i L_i(t)\}$$

$$- \Sigma_i \mu_i f^i(K_i(t), L_i(t)) + \Sigma_i \lambda_i(t) I_i(t),$$

at each date, for $L_i>0$ the time-path of the control variable L_i must satisfy, $i=1,..,N$,

$$(55) \quad w = \{p(\Sigma_i f^i(K_i(t), L_i(t)) - \mu_i e^{rt}\} f^i_L(K_i(t), L_i(t)).$$

To show that total resource extraction declines over time, note first that, for $I_i(=\dot{K}_i)>0$, $e^{-rt}q=\lambda_i$. Time-differentiating and substituting in the multiplier equation

$$\dot{\lambda}_i = -\{e^{-rt}p - \mu_i\}f^i{}_K$$

yields that

$$(56)\ rq = \{p - \mu_i e^{rt}\}f^i{}_K$$

(where arguments have now been left out). (55) and (56) can then be used to eliminate $p-\mu_i e^{rt}$. Time-differentiating the resulting equation yields that

$$(57)\ I_i = (rqf^i{}_{LL} - wf^i{}_{KL})\{wf^i{}_{KK} - rqf^i{}_{LK}\}^{-1}\dot{L}_i,$$

implying that $\dot{L}_i$ and I_i have the same sign. In other words, if $I_i>0$ during a specified interval of time, deposit i must display a production rate that is time-increasing. Next, time-differentiating (56), and substituting (57) into the resulting expression gives

$$(58)\ I_i = -\frac{(\dot{p} - r\mu_i e^{rt})}{(p - \mu_i e^{rt})}\left\{ f^i{}_{LL}\frac{(wf^i{}_{KK} - rqf^i{}_{LK})}{(rqf^i{}_{LL} - wf^i{}_{KL})} + f^i{}_{LK} \right\}^{-1}$$

whence a necessary condition for $I_i>0$ is that $\dot{p}>0$ (the term in the curly brackets is negative by the joint concavity of $f^i(.)$). That is, aggregate production should be declining.

Thus if one observes $\dot{R}(=\Sigma_i \dot{R}_i)\geq 0$ (or a non-increasing resource price) on a given interval of time, the preceding argument ensures that $I_i=0$, $i=1,..,N$, on this interval. A time-increasing aggregate output rate must therefore imply $\dot{L}_j\geq 0$ (but

$I_j=0$) for some deposit j on some interval of time. But in this case time-differentiating (56) implies that

$$(59)\ (\dot{p} - r\mu_j e^{rt})f^j_L + (p - \mu_j e^{rt})f^j_{LL}\dot{L}_j = 0.$$

However, $\dot{L}_j \geq 0$ together with $\dot{p} \leq 0$ violates (59). This rules out the possibility that $\dot{R} \geq 0$ on any interval of time, so the aggregate resource extraction rate must be everywhere a strictly declining function of time.[31]

Rather more intricate is the question of the time-pattern of output in each individual deposit. In general, the possibility that two individual deposit production rates display time rates of change with opposite signs cannot be ruled out. This has been demonstrated in a recent paper by Hung (1986), albeit using a model in which, implicitly, all inputs are variable.[32] Although in the present context one would expect the irreversibility constraint on capacity expansion to counteract this tendency somewhat, it remains a possible feature of an optimal programme.

C. Capital Equipment Decay

In the framework of section 2, suppose now that capital equipment evaporates at a constant percentage rate $\delta>0$.[33] Constraint (3) then changes to

$$(3') \quad \dot{K}_i(t) = I_i(t) - \delta K_i(t)$$

and the objective function (1) of section 2A is now maximized subject to (2), (3'), (4) and (5). A solution to this problem satisfies (6), (7) and (omitting time-arguments)

$$(8') \quad \dot{\lambda}_i - \delta\lambda_i = -\{e^{-rt}p(R) - \mu_i\}f_i'(K_i)$$

for i=1,..,N.

It is straightforward to show that under these conditions too the aggregate production profile is a strictly declining function of time. For suppose to the contrary that $\dot{R}\geq 0$ (that is, $\dot{p}=p'(R)\dot{R}\leq 0$) on a given interval of time. Then on (at least) a subinterval of this time-interval, $\dot{R}_j\geq 0$ for some deposit j. If deposit j is operating at less than capacity, $e^{-rt}p=\mu_j$, so that $\dot{p}>0$, contradicting the required result.

Deposit j must therefore be operating at full capacity, and to ensure that $\dot{R}_j=f'(K_j)\dot{K}_j\geq 0$, it is necessary that $I_j>0$. Since I_j is positive, $\lambda=e^{-rt}q$. Time-differentiating this and using the result in (8') gives

$$(r+\delta)q = \{e^{-rt}p - \mu_j\}f_j'(K_j)$$

on a non-degenerate interval of time. Time-differentiating this in turn yields

$$\dot{K}_j = - f_j'(\dot{p} - r\mu_j e^{rt})\{f_j''(p - \mu_j e^{rt})\}^{-1}.$$

For this to be non-negative, a necessary condition is that $\dot{p}>0$, which is a direct contradiction of the initial supposition.

The aggregate extraction rate is therefore everywhere a strictly declining function of time. However, as before there appears to be no obvious way of demonstrating that individual deposit output rates must always be (at least weakly) time-decreasing.

6. CONCLUDING REMARKS

The principal aim of this paper has been to reassess some common results in the exhaustible resources literature where the exploitation of resource deposits requires non-malleable capital equipment to be used in the process. Non-malleable capital has been defined here as equipment which, once installed, has no resale value.

Many of these common results remain - qualitatively - intact in the present context. For example, most of the results about the distortionary effects of fiscal instruments are similar to those which emerge under costless extraction or simple cost conditions. Also recognizable is the result that under isoelastic demand monopolistic and Cournot-Nash depletion profiles are excessively conservationist, with the latter tending to the competitive outcome if the number of operators is large.

A number of other results are less conventional. They include the finding that, under the assumption of concave technologies, deposits of different "quality" are exploited simultaneously for a period of time, a result that remains valid with the introduction of adjustment costs associated with capacity expansion. The resource price need not always be time-increasing; for certain periods of time it may remain constant or (if there are adjustment costs) fall over time. In addition, the novel result about the ambiguous interest rate effect on depletion emerges in a sharp form for the case of identical deposits displaying diminishing returns technologies with one non-malleable factor of production.

There are, of course, a number of weak points in the analysis of this paper. One is that - except in section 5B - the assumption is retained throughout that non-malleable capital is the only factor of production (or that the extraction technology is of a particular Leontief type).[34] In its absence, it is unclear in particular how individual deposit extraction rates move over time on an optimal programme. Another drawback is that the bulk of the comparative dynamic results have been derived under the assumption of identical deposits. It is not transparent, for example, that with heterogeneous deposits a larger interest rate would have a uniform effect (qualitatively) on the initial output rate of all the deposits.

Finally, there are a number of other omissions, including uncertainty, policy credibility questions in taxation, and dominant firm equilibria. A detailed treatment of these aspects would undoubtedly yield a number of interesting results, but does not appear to be an easy task.

APPENDIX A

This Appendix verifies the claims in section 2A that under the assumptions there (i) gradual capacity expansion in any given deposit j cannot begin at the initial date; (ii) in the special case where deposits are identical in every respect, gradual capacity expansion can be ruled out altogether; and (iii) any discrete addition to the stock of capital equipment in deposit j, j=1,..,N, can occur only at the initial date.

(i) Suppose that for a given deposit j, I_j is a singular control, so $I_j>0$ on some interval (t_1,t_2). Then during this interval $e^{-rt}q=\lambda_j$. Differentiating with respect to time and using (8) to eliminate $\dot{\lambda}_j$ yields that

$$rq = \{p(R) - \mu_j e^{rt}\}f_j'(K_j).$$

Time-differentiating this in turn gives (omitting arguments) an explicit expression for I_j:

$$\text{(a.1)} \quad I_j = -\frac{f_j'\{\dot{p} - r\mu_j e^{rt}\}}{f_j''\{p - \mu_j e^{rt}\}}$$

To ensure that $I_j>0$, as supposed, it is necessary that $\dot{p}>0$ on (t_1,t_2). Now $R_i(t)$ is piecewise continuous, all i=1,..,N. So it is possible to find a deposit $h\neq j$ and associated time interval (t_1,t_2^h), $t_2^h\leq t_2$, on which $R_h(t)$ is continuous with $\dot{R}_h(t)<0$ and $0<R_h<f_h(K_h)$. Then from the optimality condition (6),

$$p(R) = \mu_h e^{rt}$$

on (t_1,t_2^h). It can now be shown under the present assumption that $K_i(0)=0$, $i=1,..,N$, that gradual capacity expansion in any deposit can only begin <u>after</u> the initial date. Thus if, for an arbitrary deposit j, $I_j>0$ on some arbitrary interval of time (t_1,t_2), then $t_1>0$. For suppose that $t_1=0$, so that $e^{-rt}p(R)=\mu_h$ on $(0,t_2^h)$, $t_2^h>0$. Since optimality rules out any increase in p(R) of more than r per cent (this is shown later in section 2A), $e^{-rt}p(R)\leq\mu_h$ for <u>all</u> $t>0$. Thus, whatever the ostensible equilibrium choice of capacity for deposit h at the initial date,

$$\lambda_h(0) = \int_0^{T_h} \{e^{-rs}p(R) - \mu_h\}f_h'(K_h)ds = 0 < q \ ,$$

where T_h is the date at which extraction from deposit h stops definitively. This implies that excessive extractive capacity has been installed at deposit h. Unless inherited (that is, initial) capacity in deposit h is "very large" (by assumption it is not), this cannot feature in an optimal programme. Consequently the supposition that $t_1=0$ contradicts optimality. The initial date cannot mark the beginning of an interval on which $I_j>0$ for some deposit j.

It follows from this discussion that there must be an initial phase on which all deposits are operated at capacity (positive for at least one deposit) and no (gradual) expansion takes place. The optimal programme thus always features an initial phase on which $\dot{p}=0$.

(ii) In the special case where deposits are identical in every respect (that is, $f_i(.)=f(.)$ and $S_i(0)=S_0$, $i=1,..,N$), gradual capacity expansion can be ruled out altogether. This is done by showing that if the first observed interval of capacity expansion does not involve a capital stock discontinuity, the programme must be inefficient. Any capital stock discontinuity (other than at the initial date) is then ruled out in part (iii) of this Appendix. This implies that the first phase of investment is always inefficient, and will therefore not be observed.

Let $t_1(>0)$ denote the earliest date at which a phase of (gradual) expansion begins in at least one deposit. Pick one such deposit, and call it deposit N. Because $I_N>0$ for a phase beginning at t_1, (a.1) indicates that $\dot{p}>0$ during this phase. Let (t_1,t_2^M) denote an initial portion of this phase, and suppose that M deposits ($1 \leq M \leq N-1$) exhibit a positive but declining extraction rate during this interval of time. Call these deposits 1,..,M. Then on (t_1,t_2^M) $e^{-rt}p(R)=\mu_m$, m=1,..,M. Since $e^{-rt}p(R)>\mu_N$ over the same period, it follows that $\mu_N<\mu_m$, m=1,..,M. Moreover, because $\dot{p}>0$ on (t_1,t_2^M), $\Sigma_m\dot{R}_m(t)+\dot{R}_N(t)<0$.

Now since $e^{-rt}p(R)=\mu_m$ on (t_1,t_2^M), we deduce that $e^{-rt}p(R)\leq\mu_m$ for all $t>t_1$, m=1,..,M. This is because any price increase of more than r per cent cannot be consistent with optimality (see section 2A). Using this fact, we have from condition (8) that

$$\text{(a.2)}\quad \lambda_m(0) = \int_0^{\tau_m} \{e^{-rs}p(R) - \mu_m\}f'(K_m^+(0))ds$$

$$= \int_0^{t_1} \{e^{-rs}p(R) - \mu_m\}f'(K_m^+(0))ds$$

for deposits m=1,..,M. K_m is constant over $(0,t_1)$ because by hypothesis t_1 is the first date at which a phase of positive investment starts (we exclude the possibility of a discontinuous increase in K_m after the initial date; section (iii) below demonstrates that this cannot arise). Similarly, for deposit N

$$\text{(a.3)}\quad \lambda_N(0) = \int_0^{t_1} \{e^{-rs}p(R) - \mu_N\}f'(K_N^+(0))ds$$

$$+ \int_{t_1}^{\tau_N} \{e^{-rs}p(R) - \mu_N\}f'(K_N(s))ds.$$

Now optimality requires that $\lambda_N(0)=\lambda_m(0)=q$, $m=1,..,M$. That is, capacity buildup at the initial date proceeds up to the point where in each deposit the incremental benefit from installing a further unit then just equals the incremental cost of doing so. Equating (a.2) and (a.3), and recalling that $\mu_N<\mu_m$, $m=1,..,M$, and that $f''(.)<0$, it emerges that $K_N^+(0)>K_m^+(0)$, $m=1,..,M$. Thus, capacity in deposit N is larger on $(0,t_1)$ <u>and</u> expands further after t_1.

Next, we note that so long as deposit m ($m=1,..,M$) has not definitively ceased production, $e^{-rt}p(R)=\mu_m$. So throughout the life of deposit m, $e^{-rt}p(R)>\mu_N$, and deposit N therefore operates at full capacity. Because deposit N has larger capacity than deposit m and is operated at full capacity throughout the life of deposit m, and because deposits have identical initial stocks, it follows that: (a) deposit N is exhausted before deposit m ($m=1,..,M$); and (b) deposit N is exhausted at full capacity. Figure a.1 depicts a possible path for the stock of capital equipment in deposit N. (t_1,t_2) denotes the first expansionary phase (further expansionary phases may occur, though Figure a.1

does not show this). τ_N is the exhaustion date for deposit N.

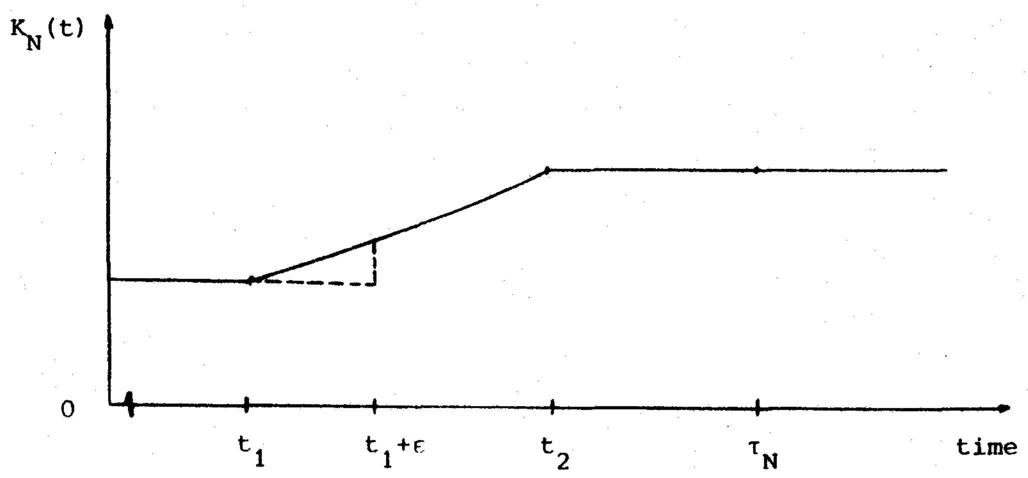


FIGURE a.1

The inefficiency can now be demonstrated by considering the following perturbation:

$K_N'(t) = K_N(t) \quad 0 \leq t \leq t_1,\ t \geq t_1+\varepsilon$; and

$K_N'(t) = K_N(t_1) \quad t_1 < t < t_1+\varepsilon$,

where $K_N'(t)$ denotes the perturbed profile and ε is an arbitrarily small positive number. In Figure a.1 this perturbation amounts to deviating from the original profile where indicated by the broken path. Since r>0, this delay in expansion entails a cost saving. It remains to demonstrate that by a suitable rearrangement of individual deposit production rates the aggregate extraction profile can be kept unchanged.

For $t \in (t_1, t_1+\varepsilon)$, there is now a shortfall of magnitude $\{f(K_N(t))-f(K_N'(t))\}$ in the contribution of deposit N to total output. By the same token, deposit N retains a portion of stock given by

$$(a.4)\quad \int_{t_1}^{t_1+\epsilon} \{f(K_N(t))-f(K_N'(t))\}dt$$

that would otherwise have been extracted on $(t_1, t_1+\epsilon)$ and will now remain at τ_N. Now make up for the output shortfall from deposit N by having a slightly larger output rate from deposit m (m=1,..,M) on $(t_1, t_1+\epsilon)$. This compensation mechanism is clearly feasible, since the requirement is that $\Sigma_m \dot{R}_m(t)+\dot{R}_N(t)<0$ on $(t_1, t_1+\epsilon)$. Deposit N will now have incurred a total reserve "debt" to deposits 1 to M given by (a.4). "Repayment" can occur on an interval of time beginning at τ_N (for sufficiently small ϵ, deposit m, m=1,..,M, will remain unexhausted at τ_N). Output from deposit m (m=1,..,M) can be reduced by an appropriate amount on this interval of time, and the shortfall made up by a positive production rate from deposit N until the portion of stock given by (a.4) has been used up entirely. At that point, deposit m (m=1,..,M) will be left with precisely the same reserves that it would have had in the original programme.

To sum up: it has been shown that the first observed phase of expansion must, if the programme is free of discontinuous increases in the stocks of capital equipment after the initial date, imply that the allocation is inefficient. In other words, without attendant jumps in capital equipment stocks, the earliest phase of capacity expansion must be inefficient, and thus would never be observed in an optimal programme. The final section of this Appendix now shows that the strict concavity property of the optimization problem precludes discontinuous increases in capital stocks after the initial date.

(iii) This section establishes that any discontinuous additions to capital equipment must all occur at the initial date t=0. Note first that the objective function (1) can be written

$$\text{(a.5)} \quad \int_0^{\infty} e^{-rt}\{ u(\Sigma_i R_i(t)) - q\Sigma_i I_i \} dt - q \sum_{g=1}^{G} \exp(-r\tau_g) \sum_{i=1}^{N} \{K_i^+(\tau_g) - K_i^-(\tau_g)\} .$$

(a.5) is optimized subject to constraints (2), (3) and

$$\text{(a.6)} \quad K_i(t) = \sum_{\tau_g < t} \{K_i^+(\tau_g) - K_i^-(\tau_g)\} + \int_0^t I(s)ds$$

where $K_i^+(\tau_g)-K_i^-(\tau_g)\geq 0$, $i=1,..,N$ and $g=1,..,G$. The date τ_g ($g=1,..,G$) represents a jump date when at least one of the K_i increases discontinuously. The timing of each jump date and the magnitude of the jump in each variable at each such date (perhaps zero) must be determined optimally. $I_i(t)$ now assumes only finite non-negative values, $i=1,..,N$.

As shown by Vind (1967), Arrow and Kurz (1970, pp. 51-7) and Kamien and Schwartz (1981, pp. 226-32), this problem has the following equivalent: choose the time paths of $R_i(w)$, $I_i(w)$ and $J_i(w)$, $i=1,..,N$, and $v_0(w)$ to maximize

$$\text{(a.7)} \quad \int_0^{\infty} e^{-rt} v_0(w)\{u(\Sigma_i R_i(w)) - q\Sigma_i I_i(w)\} - e^{-rt}(1-v_0(w))q\Sigma_i J_i(w)dw$$

subject to the constraints

$$\text{(a.8)} \quad \frac{d}{dw} S_i(w) = - v_0(w)R_i(w), \; S_i(0) = S_{i0}, \; \lim_{w\to\infty} S_i(w) \geq 0;$$

(a.9) $\frac{d}{dw} K_i(w) = v_0(w) I_i(w) + (1 - v_0(w)) J_i(w), \; K_i(0) = 0;$

(a.10) $v_0(w)\{f_i(K_i(w)) - R_i(w)\} \geq 0;$

(a.11) $v_0(w) R_i(w) \geq 0;$

(a.12) $v_0(w) I_i(w) \geq 0;$

(a.13) $(1 - v_0(w)) J_i(w) \geq 0;$

all for i=1,..,N, and

(a.14) $\frac{dt}{dw} = v_0(w) = \begin{cases} 0 & \text{during a jump interval} \\ 1 & \text{otherwise.} \end{cases}$

Notice that jumps in the state variables $K_1, .., K_N$ no longer appear in the transformed problem. The basic idea is that the problem is reformulated in "artificial time" w, which runs apace with natural time t between jumps (equation (a.14)). At a jump date, however, the passage of natural time is halted, but artificial time continues to run for a specified interval, during which a jump in K_i, if required, can be effected smoothly for i=1,..,N (a dummy control, J_i, simply assumes a positive value over the interval, or a value of zero if no jump is desired).

Let μ_i and λ_i, i=1,..,N be the multiplier functions corresponding to (a.8) and (a.9) respectively, and λ_0 the multiplier function corresponding to (a.14). The present value Lagrangian for (a.7) - (a.14) reads

(a.15) $L = v_0(w) \{ e^{-rt} [u(\Sigma_i R_i(w)) - \Sigma_i I_i(w)]$

$+ \lambda_0(w) - \Sigma_i \mu_i R_i(w) + \Sigma_i \lambda_i(w) I_i(w)$

$$+ \Sigma_i \alpha_i(w)\,[f_i(K_i(w)) - R_i(w)] + \Sigma_i \beta_i(w) R_i(w)$$

$$+ \Sigma_i \gamma_i(w) I_i(w) \;\}$$

$$+ (1-v_0(w))\;\{\; \Sigma_i \lambda_i(w) J_i(w) - e^{-rt} q \Sigma_i J_i(w)$$

$$+ \Sigma_i \delta_i(w) J_i(w) \;\}$$

$$= v_0(w)\, L_a + (1-v_0(w)) L_b \quad \text{say,}$$

where α_i, β_i, γ_i and δ_i, i=1,..,N are the Lagrange multiplier functions appended to (a.10), (a.11), (a.12) and (a.13). Now let L^0 denote the maximum value of L. Then

$$L^0 = \max_{v_0} \{v_0(w) L^0_a + (1-v_0(w)) L^0_b\}$$

where

$$L^0_a = \max_{\substack{\{R_1,..,R_N\}\\ \{I_1,..,I_N\}}} L_a$$

and

$$L^0_b = \max_{\{J_1,..,J_N\}} L_b$$

Now if J_i, i=1,..,N is chosen to be some non-negative function over a jump interval (when L^0 takes the value L^0_b), another function can always be found that is constant over the jump interval and yields the same value of (a.7). So without loss of generality J_i, i=1,..,N can be viewed as a constant function (perhaps zero) over each jump interval.

Next, it should be noted that $\lambda_i(w) \leq e^{-rt} q$ for all i, w. Failing this, J_i could be set arbitrarily high and (a.7) would have no maximum. So $L^0_b \leq 0$, and in fact $L^0_b = 0$ can always be obtained by setting $J_i = 0$, i=1,..,N. Therefore

$$L^0 = \max_{v_0} v_0(w) L^0_a$$

from which it is evident that $L^0_a \leq 0$ if $v_0(w)=0$ whereas $L^0_a \geq 0$ if $v_0(w)=1$. Consider now a hypothetical jump date $\tau_g > 0$ (that is, not the initial date) in natural time, and denote the associated jump interval in artificial time by (w_g^-, w_g^+). Since L^0 is continuous in w, and zero on a jump interval, L^0_a must be zero both at the beginning and at the end of this jump interval (and naturally $L^0_a \leq 0$ during the interval).

It is however straightforward to verify that on the proposed jump interval (w_g^-, w_g^+), L^0_a must be strictly concave in w, in direct contradiction to the requirement in the preceding sentence. The argument is as follows. Firstly, each K_i is linear in w ($J_i \geq 0$ is constant by construction). Secondly, $\lambda_0(w)$ satisfies

$$\frac{d\lambda_0(w)}{dw} = - L_t = r\, v_0(w)\, e^{-rt}[u(\Sigma_i R_i(w)) - q\Sigma_i I_i(w)]$$

$$- r\, (1-v_0(w))\, e^{-rt} q\Sigma_i J_i(w)$$

$$= -r\, e^{-rt}\, q\Sigma_i J_i(w) \quad \text{on a jump interval,}$$

so that it is also linear in w on such an interval. Finally, from (a.15), and letting $R_i{}^*$ and $I_i{}^*$ denote the choice of R_i and I_i, $i=1,..,N$ that maximizes L_a, we have

$$\text{(a.16)} \quad \frac{d^2 L^0_a}{dw^2} = e^{-rt}\, [u''(\Sigma_i R_i{}^{*\prime})^2 + u'\Sigma_i R_i{}^{*\prime\prime}$$

$$- q\Sigma_i I_i{}^{*\prime\prime}] + \lambda_0{}'' - \Sigma_i \mu_i R_i{}^{*\prime\prime}$$

$$+ \Sigma_i \{2\lambda_i{}' I_i{}^{*\prime} + \lambda_i{}'' I_i{}^* + \lambda_i I_i{}^{*\prime\prime}\}; \text{ or}$$

$$\text{(a.16')} \quad \frac{d^2 L^0_a}{dw^2} = e^{-rt}\, u''\, (\Sigma_i R_i{}^{*\prime})^2 + \Sigma_i R_i{}^{*\prime\prime}\, \{e^{-rt} p - \mu_i\}$$

where the arguments of u'' and $u'=p$ have been omitted. The terms in I_i^* drop out because for $I_i^*>0$, $\lambda_i=qe^{-rt}$ remains constant over the jump interval.

The first term on the right-hand side of (a.16') is non-positive, since $u''<0$ and $(\Sigma_i R_i^{*\prime})^2\geq 0$. Consider now the second term, component by component. Clearly if $(e^{-rt}p-\mu_i)\leq 0$, then

$$R_i^{*\prime\prime}(e^{-rt}p-\mu_i) = 0$$

(recalling that $e^{-rt}p-\mu_i<0$ implies $R_i^*=0$). On the other hand, if $e^{-rt}p-\mu_i>0$, then

$$R_i^{*\prime\prime}(e^{-rt}p-\mu_i) = f_i''(K_i(w))[K_i'(w)]^2(e^{-rt}p-\mu_i) \leq 0,$$

with strict inequality if $J_i>0$ on the jump interval (recall that $e^{-rt}p-\mu_i>0$ implies $R_i^* \equiv f_i(K_i(w))$, and that by construction $K_i(w)$ is linear in w on a jump interval). Finally, by definition $J_j>0$ for some deposit j on a jump interval, and it is clear that for this deposit $e^{-rt}p-\mu_j>0$ during the interval. If instead $e^{-rt}p\leq\mu_j$ at any time during the interval, this must remain the case for all (natural and artificial) time subsequently, because p cannot grow at a rate exceeding r (see section 2A). This means that $\lambda_j=0$, so additional capital equipment in deposit j is worthless, which contradicts the supposed optimality of setting $J_j>0$ during the interval.

Thus at least one term in (a.16') is negative, so $d^2L^0{}_a/dw^2<0$ and $L^0{}_a$ is strictly concave in w, as asserted earlier. This rules out a jump in K_i, i=1,..,N, at any date other than the initial date.

APPENDIX B

This Appendix details the steps in the derivations used in section 4B of the text.

(i) To obtain expression (38) for V^M_K from (36'):

Differentiate (36') with respect to K. This gives

$$V^M_K = N\Big[\ (1-(1+rT)e^{-rT})\Big\{\frac{p(Nf(K))+p'(Nf(K))Nf(K)}{r}\Big\}f'(K) + p(Nf(K))f(K)rTe^{-rT}\frac{dT}{dK} + e^{-rT}\,S_o\,\Big\{p'(Nf(K))Nf'(K) - rp(Nf(K))\frac{dT}{dK}\Big\}\ \Big].$$

Some rearrangement yields

$$\text{(b.1)}\quad V^M_K = N\Big[\ (1-(1+rT)e^{-rT})p(Nf(K))f'(K) + (1-e^{-rT})p'\frac{(Nf(K))Nf(K)f'(K)}{r} + e^{-rT}(S_o-Tf(K))\Big\{p'(Nf(K))Nf'(K) - rp(Nf(K))\frac{dT}{dK}\Big\}\ \Big].$$

Now dT/dK is given by equation (16) (section 2C) in the text. Put this into (b.1) to evaluate the term in curly brackets. (b.1) then becomes

$$\text{(b.2)}\quad V^M_K = N\Big[\ (1-(1+rT)e^{-rT})\,\frac{p(Nf(K))f'(K)}{r} + (1-e^{-rT})\,\frac{p'(Nf(K))Nf(K)f'(K)}{r} - \frac{e^{-rT}\,(S_o-Tf(K))NTf'(K)}{\int_T^{\infty} D'(p(Nf(K))e^{r(t-T)})e^{r(t-T)}dt}\ \Big].$$

Next, multiply and divide the second and third terms on the right-hand side of (b.2) by p(Nf(K)) and use the facts that

$$\text{(b.3)} \quad S_0 - Tf(K) = \frac{1}{N}\int_T^\infty D(p(Nf(K))e^{r(t-T)})dt; \text{ and}$$

$$\text{(b.4)} \quad \frac{p(Nf(K))}{p'(Nf(K))Nf(K)} = \frac{D'(.)p(Nf(K))e^{r(t-T)}}{D(.)} = \eta \quad \text{for } t \geq T$$

where the argument of D(.) and D'(.) is understood to be $p(Nf(K))e^{r(t-T)}$. A few cancellations now give

$$\text{(b.5)} \quad V^M_K = N\Big[\,(1-(1+rT)e^{-rT}\frac{p(Nf(K))f'(K)}{r}$$

$$+ \; (1-e^{-rT})\frac{p(Nf(K))f'(K)}{r\eta}$$

$$- \; e^{-rT}\,T\,\frac{p(Nf(K))f'(K)}{\eta}\Big].$$

Equation (38) in the text then follows directly from (b.5).

(ii) To obtain expression (39) for V^M_K in the text:

For $\eta'(R)<0$, the derivative V^M_K reads as in the isoelastic case up to (b.2). Using (b.3), the last term in (b.2) becomes

$$\text{(b.6)} \quad \frac{-e^{-rT}\int_T^\infty D(p(Nf(K))e^{r(t-T)})dt\;Tf'(K)}{\int_T^\infty D'(p(Nf(K))e^{r(t-T)})e^{r(t-T)}dt}$$

(omitting the factor of N outside the square brackets in (b.2)). Now multiply and divide the integrand in the denominator by $D(p(Nf(K))e^{r(t-T)})$, and (b.6) itself by p(Nf(K)). (b.6) then becomes

$$\frac{-e^{-rT}\int_T^{\infty} D(p(Nf(K))e^{r(t-T)})dt \; Tf'(K)p(Nf(K))}{\int_T^{\infty} D(p(Nf(K)e^{r(t-T)})\eta(D(p(Nf(K))e^{r(t-T)}))dt}$$

or, using the definition of G(T) in the text (see equation (39)),

$$\text{(b.7)} \quad \frac{-e^{-rT}\int_T^{\infty} D(p(Nf(K))e^{r(t-T)})dt \; Tf'(K)p(Nf(K)).}{G(T)\;\eta(Nf(K))\int_T^{\infty} D(p(Nf(K))e^{r(t-T)})dt}$$

Finally, using (b.7) to simplify (b.2) gives (39).

(iii) To obtain the expression (43) for V^M_K:

For $\eta'(R)>0$, (41) gives the maximum value function. Its derivative with respect to K is

$$\text{(b.8)} \quad V^M_K = N\Big[\ \frac{(1-e^{-rT})}{r}(f(K)p'(Nf(K))Nf'(K) + p(Nf(K))f'(K)) + e^{-rT}\,p(Nf(K))f(K)\frac{dT}{dK}$$

$$+ e^{-rT}\{m'(Nf(K))Nf'(K) - rm(Nf(K))\frac{dT}{dK}\}\int_T^{\infty} n'(t)\{p(t)+p'(t)n(t)\}dt\ \Big],$$

where

$n'(t) = n'(m(Nf(K))e^{r(t-T)})$

$n(t) = n(m(Nf(K))e^{r(t-T)})$

$p(t) = p(n(t))$ and

$p'(t) = p'(n(t))$.

Now from (42), which is the appropriate reserve constraint here (b.9) $$\frac{dT}{dK} = \frac{TNf'(K) + Nf'(K)m'(Nf(K))\int_T^\infty e^{r(t-T)}n'(t)dt}{r\, m(Nf(K))\int_T^\infty e^{r(t-T)}n'(t)dt}$$

Using (b.9), the term in curly brackets in (b.8) can be simplified to

$$-TNf'(K) \Big/ \int_T^\infty e^{r(t-T)}n'(t)dt.$$

Replace this in (b.8), and note that

$$(p(t) + p'(t)n(t)) = m(Nf(K))e^{r(t-T)}.$$

Making the appropriate cancellations in (b.8) and collecting terms then gives equation (43).

APPENDIX C

This Appendix demonstrates that, in the case of identical deposits with identical capacity adjustment cost functions, the optimal programme features identical investment profiles across deposits.

Consider the objective of minimizing, by choice of $\{I_i(s)\}^t_{s=0}$, $i=1,..,N$, the PV of the costs

(c.1) $$\int_0^t e^{-rs}\Sigma_i\{qI_i + C(I_i(s))ds$$

of constructing a given aggregate output capacity $\bar{R}$ by some date $t>0$. The constraints in this optimization problem are

(c.2) $\dot{K}_i(s) = I_i(s)$, $K_i(0)=K_0$

(where K_0 is "small"), and the "terminal" condition

(c.3) $\Sigma_i f(K_i(t)) - \bar{R} = 0$

for $i=1,..,N$. A solution must satisfy, for $0 \leq s \leq t$,

(c.4) $e^{-rs}\{q + C'(I_i(s))\} + \sigma_i \geq 0$, $I_i(s) \geq 0$ (CS)

where σ_i is a constant, $i=1,..,N$. Moreover the endpoint condition (c.3), together with Farkas' lemma implies (Kamien and Schwartz, 1981, pp. 143-8) that there must exist a number α, independent of the subscript i, such that for $i=1,..,N$

(c.5) $\sigma_i = \alpha f'(K_i(t))$.

Now pick any pair of deposits, say h and j. If $K_h(t) = K_j(t)$, then (c.4) and (c.5) immediately imply that $I_h(s) = I_j(s)$ for $0 \leq s \leq t$, so the two fields must have identical associated investment profiles. It remains then to rule out the possibility that $K_j(t) \neq K_h(t)$. Suppose without loss of generality that $K_h(t) > K_j(t)$. For s such that $I_j(s), I_h(s) > 0$, (c.4) and (c.5) imply that

$$\text{(c.6)} \quad \frac{q+C'(I_h(s))}{f'(K_h(t))} = \frac{q+C'(I_j(s))}{f'(K_j(t))}.$$

So if $K_h(t) > K_j(t)$, $f'(K_h(t)) < f'(K_j(t))$, and therefore $C'(I_h(s)) < C'(I_j(s))$, implying that $I_h(s) < I_j(s)$ for any s if both I_h and I_j are positive. But then deposit h cannot have a larger "terminal" stock of capital equipment than deposit j, unless for some s', $0 < s' < t$, $I_h(s') > I_j(s') = 0$. In this case (c.4) and (c.5) together yield that

$$\frac{q}{f'(K_j(t))} \geq \frac{q+C'(I_h(s'))}{f'(K_h(t))}$$

which is a contradiction. Thus $K_j(t) = K_h(t)$ for arbitrary j and h. The "terminal" date t and the desired capacity were also arbitrary, so this holds for any $t > 0$ and $R > 0$.

Finally, since every deposit contains the same amount of the resource, a symmetric extraction profile across deposits at all dates is feasible, so the "trial" solution (identical investment profiles) is indeed an optimal programme.

NOTES

1. Increasing returns in the extraction process create difficulties for the existence of competitive equilibrium if mixed ("chattering") strategies are disallowed (Eswaran et al., 1983). Essentially the problem arises because the second-order instantaneous equilibrium conditions demand that firms remain on the upward sloping portions of their marginal cost curves, and this is inconsistent with the requirement that extraction decline smoothly to zero.

2. Depending upon the case at hand, "investment" activity can refer variously to well-drilling, mineshaft construction or platform erection as well as the purchase and installation of smaller items of plant equipment. The rationale for assuming that adjustment costs are convex then acquires a greater or lesser degree of cogency depending on the type of investment activity referred to.

3. For a multi-deposit framework, Blackorby and Schworm consider both the case where (i) all inputs used in the extraction process are variable; and (ii) fixed capital - changes in which incur adjustment costs - is used in the extraction process.

In case (i), it is possible to construct an aggregate measure of extraction (not necessarily the sum of individual deposit extraction rates) provided that at least one firm (i.e., deposit) is subject to a non-linear extraction technology. All firms facing non-linear net revenue functions must, moreover, be of the same type. If, additionally, one requires an aggregate resource stock measure - such that its time-derivative equals minus the extraction rate - then the individual net revenue functions must either all display additive separability, or all display linear homogeneity in the stock and the extraction rates. In all cases, allocation functions defined alongside determine the allotment of the aggregate magnitudes across individual deposits. Somewhat disturbingly - although it is admittedly an oversimplified case - a consistent aggregate version of the extraction problem <u>cannot</u> be constructed if individual firms' net revenue functions depend only on the current extraction rate (and not reserves).

In case (ii), it is possible to construct an aggregate measure of extraction, investment and net revenue (along with the appropriate allocation functions) provided that: (a) at least one firm has a net revenue function that is non-linear in the investment rate (and fulfils certain separability requirements) with all such non-linear firms displaying the same type of net revenue function; <u>and</u> (b) at least one firm has a net revenue function that is non-linear in the extraction rate (and meets certain separability restrictions), again with all such non-linear firms displaying the same type of net revenue function. Conditions for the existence of aggregate measures of capital and resource stocks in these cases can be similarly elucidated.

4. In many applications, this is at best an oversimplified description of the production possibility set. In the case of petroleum, for example, output possibilities at a given date depend not only on "capacity" as it is generally understood (number of wells in place, pipeline capacity, etc.) but also on reservoir pressure, which in turn depends on the precise volume and pattern of cumulative production, and, where "secondary" recovery is concerned, on the rate and timing of water injection, etc. Discussions of optimal reservoir exploitation when these attributes are taken into account appear in Kuller and Cummings (1974) and Nystad (1984). On a more general note, Levhari and Liviatan (1977) analyse the optimal use-trajectory for an exhaustible resource where extraction costs are a decreasing function of the remaining stock. They demonstrate that the rate of extraction may, over a certain interval, increase over time, a result that is in stark contrast to the usual one (derived when this effect is neglected) that extraction is a continuously declining function of time. In what follows the above considerations are ignored, but it should be noted that they could severely qualify the results.

5. Alternatively, any (internal) costs of adjustment are linear in the rate of investment and subsumed under the purchase price. Since the resale value of a unit of equipment is zero, the manager will be indifferent between removing a unit and leaving it in place once it is definitively no longer required (the unit does not <u>hinder</u> extraction) if removal costs nothing. If, on the other hand, there is a removal cost - however small - disinvestment never takes place; hence the irreversibility constraint on investment in the optimization problem (1)-(5).

6. If instead deposit 1 had some reserves left at T_2, an increase in the objective function could be achieved by extracting a little more from deposit 1 on (T_1, T_2). This is feasible, because by assumption deposit 1 operates at less than full capacity on this interval. It is also preferable, because the discounted marginal value (MV) of the resource is larger on (T_1, T_2) than after T_2 (by assumption price (MV) is constant, so the discounted price is falling, on (T_2, T_3)).

7. If we assume that total output is at each date equally distributed across fields during (T, ∞), then for no deposit is capacity a binding constraint after T. However, this is not the only possible equilibrium. For $t \geq T$, any temporal distribution of output between the two fields is acceptable provided only that the <u>aggregate</u> extraction profile declines at the appropriate rate and exhausts asymptotically the available reserves, and that the individual deposit output rates are piecewise continuous functions of time. Thus it is possible for some deposits to be operating at full capacity even though aggregate output is declining.

8. See note 7.

9. Note that, if the variable of integration is changed from t to $\varpi \equiv p(Nf(K))e^{r(t-T)}$,

$$\int_T^{\infty} D'(p(Nf(K))e^{r(t-T)})p(Nf(K))e^{r(t-T)}dt = \frac{1}{r}\int_{p(Nf(K))}^{\infty} D'(\varpi)d\varpi$$

$$= \frac{1}{r}\{\lim_{\varpi \to \infty} D(\varpi) - D(p(Nf(K)))\}$$

$$= -\frac{N\,f(K)}{r} \quad \text{(assuming } \lim_{\varpi \to \infty} D(\varpi) = 0\text{).}$$

10. This term refers to a deposit under unitary control, either as a result of historically given ownership patterns (or requirement by the licensing authority) or private contracting to overcome the "rule of capture" problem. Wiggins and Libecap (1985) find evidence that the widespread absence, historically, of oil field unitization in the United States can be attributed largely to <u>ex ante</u> imperfect information and information asymmetries regarding the value of individual operators' leases. This hinders the erection of judicious rent-sharing schemes.

11. The complete set of *forward markets* gives resource owners a complete picture of the intertemporal resource price trajectory and guarantees that, through arbitrage, the r per cent rule is satisfied and the correct initial extraction rates are determined. The requirement that the market rate of interest should equal the social discount rate is then tantamount to the requirement that the present value endowments of all consumers - present and future - should be appropriately distributed.

12. If, as the extraction programme unfolds, only a small number of operators remain active after some date, the price-taking assumption may no longer be viable, so the optimality properties of what began as a competitive programme may break down. This possibility is not taken up here. Partly it is no easy matter to rationalize the exact point at which competition breaks down; partly also operators will be aware of the uncompetitive tendency from the initial date and may be able to negate the effect through intertemporal arbitrage.

13. See note 7.

14. As pointed out by Gamponia and Mendelsohn (1985), what matters more than the direction of bias is the magnitude of the welfare loss implied by the tax and, in particular, the welfare loss implied by one tax versus another (assuming both are designed to raise a given revenue). Using a model in which the only costs are constant unit operating costs, Gamponia and Mendelsohn simulate numerically, for different parameter values, the effects of various distortionary taxes on the discounted stream of consumer plus producer surplus. Their main finding is that a tax on gross revenue entails the smallest loss.

15. This can be deduced using broadly the same method as that used to verify the latter part of propositions 1 and 2.

16. If, however, the initial resource stock available to the operator is a choice variable (e.g., the operator can choose how much to spend on locating additional reserves) the tax is no longer a pure rent tax and induces a reduction in the total stock extracted. This is shown by Gaudet and Lasserre (1984). If, of course, exploration costs are fully offset against taxable income, the neutrality of the rent tax is restored. However, as Campbell and Lindner (1985) point out, under uncertainty about the value of a deposit and with risk averse operators, a higher rent tax rate may actually increase the ex ante probability of exploitation following exploration (a rent tax fulfils the function of a risk-sharing scheme) if the resource deposit is viewed initially as being sufficiently promising or sufficiently unpromising.

17. Strictly speaking, this equilibrium is sustainable only if the resource is instantly perishable. Suppose, to take the opposite extreme, that the resource can be stored costlessly. Then the resource price cannot rise by more than r per cent. But then the resource owners cannot be induced to hold on to the resource after T (because the present value of marginal profit is declining over time), i.e., they wish to continue producing at full capacity after T. At the same time, if all resource owners depleted at maximum capacity, exhaustion would occur in finite time, and willingness to pay for the resource at the margin would be infinite. So the conditions required for equilibrium seem inconsistent with one another.

18. Although propositions 1 and 2 (section 2C) tell us that, to the extent that operators respond to risk in future returns by increasing their discount rates, the effect on the pace of depletion is ambiguous.

19. Naturally the multiplier functions will differ from their analogues in the previous sections. The same symbols are used throughout to economize on notation.

20. The analysis is confined to the elastic portion of the demand curve where marginal revenue is positive. If demand is everywhere inelastic, the monopolist does best by supplying an arbitrarily small amount of the resource at each date (see Tullock, 1979). It may be objected that - for the current range of prices - the demand for many exhaustible resources (oil in particular) is inelastic, and yet we observe positive output by sellers with putative monopoly power. There are a number of explanations for this, including the presence of fringe resource suppliers, conventionally produced substitutes that put a ceiling on the resource price, and elastic long-term demand responses.

21. This sentence needs severe qualification: strictly speaking it is correct only where there is a single deposit and a fixed cost must be borne but there is no capacity constraint of any

kind on extraction. With more than one deposit a) deposits are exploited sequentially; b) competitive exploitation does not in general reproduce the socially optimal outcome; and c) monopoly exploitation is too conservationist (Hartwick et al., 1986).

22. $m'(R) \leq 0$ is in any case a second-order necessary (Legendre) condition for an interior solution. Note that this does not necessarily interfere with the existence of an interior maximizing solution in case (ii) ($\eta'(R)<0$); see for example Lewis et al. (1979, footnote 8).

23. Note that for the present case ($\eta'(R)>0$) and the previous case ($\eta'(R)<0$) the assumption that there is no chokeoff price - so that the resource is exhausted only asymptotically - has been retained for convenience. As an example, an inverse demand function of the form $p(R) = A \log (B/R)$ (defined for $R \leq B$), where A and B are positive constants, displays $\eta'(R)>0$, while $p(R) = A \log (B/R) + (C/R)$ where C is also a positive constant, displays $\eta'(R)<0$ for all R below a certain output. Neither form has a chokeoff price. Another example appears in Lewis et al. (1979, footnote 8).

24. Other equilibrium concepts, such as those based on Bertrand conjectures or consistent conjectures, do not appear to have been investigated in a resource extraction context.

25. The Nash-Cournot equilibrium appears to offer a close approximation to the perfect equilibrium, which except in simple cases is exceedingly difficult to derive analytically (see Newbery, 1981, pp. 633-5).

26. A symmetric distribution of output across operators for $t>T$ (i.e, once output begins to decline) is the only solution consistent with equilibrium. Temporal changes in the share of individual firms in total output, though possibly consistent with a perfectly competitive market structure, raise a contradiction here. Given that deposits are identical in all respects, we require that $\mu_h = \mu_j$, all h,j (the marginal value of a unit of stock in situ should be the same for every firm). From equation (44), we then have

$$p(1+v_h/\eta) = p(1+v_j/\eta)$$

for all h,j (operating at less than full capacity), where $v_i = R_i/\Sigma R_i$. We therefore require $v_h(t) = v_j(t)$ for all t. (Note that if field h (say) operates at full capacity after T while field j does not, this implies that $v_h < v_j$, which means that field h has smaller capacity and contradicts the requirement of identical unit rents).

27. Phases on which $I_i>0$ but $R_i<f(K_i)$ for deposit i are not observed. To see this, note that $R_i<f(K_i)$ implies $e^{-rt}p(R) \leq \mu_i$. Since p cannot grow by more than r per cent on any interval of time, $e^{-rt}p(R) \leq \mu_i$ at all dates subsequently. Combining this information with the boundary condition on λ_i, one infers that λ_i

must be zero on the proposed phase (and thereafter). But this means that additional capital equipment in deposit i is worthless.

28. If, for example, deposits are identical in all respects, an optimal programme features identical capacity buildup across deposits (see Appendix C) and thus identical extraction rates so long as all deposits are worked at maximum capacity. However, once the (final) phase of r per cent price growth begins, extraction rates need no longer be identical across deposits. Aggregate extraction must be time-decreasing, but this does not preclude one deposit continuing to operate at capacity to exhaustion, if extraction from at least one other deposit declines sufficiently rapidly and exhibits a discontinuous increase in output once the first is exhausted.

29. The table below gives an indication of past and projected future output profiles for UK North Sea Oilfields. While there is no such thing as a "representative" output profile for an oilfield, very loosely speaking one can identify initial phases of increasing output (during which "investment", including exploration, is still being carried out), followed by phases of roughly constant output and then of diminishing output. Although one cannot claim that the analysis of this section is precisely applicable, the figures may be seen as suggestive. On a more rigorous note, a study by Lasserre (1985b), using data for a sample of North American non-ferrous mines, found capacity choice (the erection of a new operation or significant expansion of an existing one) to be explained quite well in terms of price and deposit data, beginning from the type of approach discussed in this paper.

30. In general, capacity expansion may continue in other deposits during this phase of r per cent price growth.

31. However, the constant extraction result obtained in the single (fixed capital) input case is preserved here under a Leontief technology of the form

$$R_i \leq \min\{f_1^i(K_i), f_2^i(L_i)\}$$

where f_2^i displays constant returns and is identical across deposits. In this case, capital merely determines the upper bound on feasible output: it does not otherwise affect the productivity of the variable input. Moreover successive units of the variable factor are equally productive irrespective of which deposit they are allocated to .

Where the technology given by (4') is concerned, Lewis (1985) has pointed out that if capital imposes an upper bound on production in the sense that $f^i(K_i, L_i)$ is maximized, for given K_i, by some finite $L_i{}^*(K_i)$, then "capacity" in this sense will never be fully employed. The reason is that at $L_i{}^*$ the marginal product of L in deposit i would be driven to zero, contradicting condition (55).

Actual and Projected Output Rates for U.K. North Sea Oil Fields
('000 Barrels/Day)

Field	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Beryl	-	8	61	54	94	110	97	92	80	89	102	110	115	118	122	125	120	110	88	79	71
Brent	-	3	26	77	179	140	230	322	401	427	474	480	440	400	360	323	270	235	205	178	158
Claymore	-	-	6	59	80	88	90	94	93	100	85	70	53	44	38	32	30	26	24	21	19
Cormorant	-	-	-	-	1	22	18	44	78	128	150	180	178	171	148	125	105	92	76	63	51
Forties	13	178	421	513	515	522	486	476	463	416	381	310	264	235	161	110	75	55	45	30	25
Fulmar	-	-	-	-	-	-	-	56	119	126	110	140	150	150	120	90	75	65	50	35	25
Ninian	-	-	-	1	158	231	293	307	280	237	223	195	175	125	100	80	75	70	65	60	55
Piper	-	2	176	254	274	218	212	200	199	182	185	180	145	100	75	55	45	40	25	20	-

Source: Wood Mackenzie & Co., North Sea Reference Section, August 1986.

32. Hung constructs the following example for N=2. For deposit 1 extraction costs are linear and given by c_1R_1, while for deposit 2 they are given by $C_2(R_2)$, with $C_2'(.) > 0$, $C_2''(.) > 0$ and $C_2'(0) > c_1$. The maximand is, as usual, the stream of consumer plus producer surplus, discounted at the appropriate rate r.

To begin with, it can be demonstrated that, if any ε-interval contains a strict sequencing of extraction (that is, the interval contains a switch from production in deposit 1 only to production in deposit 2 only, or vice-versa), then reallocating production over the interval to achieve simultaneous exploitation yields an increase in the objective function. This holds so long as one deposit at least displays a strictly convex cost function (diminishing returns to scale). So a non-degenerate interval of time can be found during which output from both deposits is positive. Next, on this latter interval

$$e^{-rt}\{p(R_1+R_2) - c_1\} = \mu_1$$

$$e^{-rt}\{p(R_1+R_2) - C_2'(R_2)\} = \mu_2$$

where μ_1 and μ_2 are constants (the present value rents). Eliminating p(.) between these two equations yields that

$$c_1 + \mu_1 e^{rt} = C_2'(R_2) + \mu_2 e^{rt}, \quad \text{or}$$

$$r(\mu_1 - \mu_2)e^{rt} = C_2''(R_2)\dot{R}_2$$

by time-differentiation. Finally, we require $\mu_1 > \mu_2$ (the high cost deposit has a lower rent, otherwise it can be shown to remain in the ground forever, which is suboptimal), so that $\dot{R}_2 > 0$. Needless to say total output of the resource must nonetheless be declining over this interval.

33. Naturally the validity of the result that follows is restricted to the case where decay is both mechanistic and continuous. If, for example, a given piece of plant equipment either works or does not, and the probability of failure depends upon maintenance/replacement expenditure (and possibly the intensity of use), one would expect still to observe phases during which total production of the resource is constant.

34. See note 31.

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